Introduction

It would be easy to get the impression that there’s a huge gap between theory and practice in university mathematics teaching. On the one hand, educational research is widely portrayed as having exploded the myth that lecturing is an effective form of teaching. On the other hand, lectures remain a staple of most degree courses, and even seem to be reasonably popular with students studying mathematics [1, 2]. Are lecturers ignoring important evidence that could improve their teaching? Are educational researchers living in a fantasy world with no connection to the chalkface? Or can theory and practice, perhaps, be reconciled?

In a recent review [3], I have argued that the lecture, deployed as part of a sensible teaching strategy, still has much to offer and can be defended in the light of educational research. In this article, I wish to outline one way of looking at the functions that lectures can perform effectively. This account is certainly incomplete, but it suggests that it is possible to continue lecturing in university mathematics with some confidence that our students can benefit from it.

Underlying assumptions and arguments against lecturing

In this article, I will not discuss in detail the flaws that I believe exist in many arguments against lecturing (for more discussion see [3]). It is important, though, to note a couple of points which are sometimes overlooked.

First, when importing findings from other fields, we must remember that mathematics, as the extreme ‘hard pure’ discipline, is an anomalous subject [4] (and mathematicians are often anomalous creatures). We tend to be aware that our subject is unusually strongly structured and objective. We may be less aware that we deploy lectures differently from many other disciplines, and that in particular the lecture-homework-tutorial cycle is rarely considered in discussions of lecturing. Even leaving aside issues such as publication bias [5], then, it is clear that the findings of surveys such as Bligh’s “What’s the Use of Lectures?” [6] will require careful interpretation for mathematics.

Second, it is common but dangerous to assume that students are essentially rational [7]. This is usually cast as the assumption that students will implement a learning strategy based on the teaching and assessment they experience and on their own underlying priorities — perhaps maximising learning; perhaps minimising effort while passing the course [8]. Under this assumption, they should respond well to a regime of self-directed learning opportunities and assessment based on deep
learning. Students, however, are human, and may act fervently for, or diligently against their own interests, so teaching must address ‘affective’ factors such as habit, enthusiasm and identity, as well as ‘cognitive’ factors related to learning. A strength of lectures is that they can address these factors simultaneously.

Three overlapping roles of lectures

I argue that lectures can perform several functions well. The complicated nature of learning means that these functions overlap, and in fact they are all facets of a single basic idea: “in lectures, students are invited to enter the lecturer’s world, guided by their experience and familiarity with the territory” [9].

The three functions I will outline are:

- **Communicating** information, definitions, theorems, methods and overviews;
- **Modelling** problem-solving, heuristic and formal reasoning, and ‘expert’ thought processes; and
- **Motivating** students to approach the subject with an appropriate attitude and enthusiasm.

Figure 1 illustrates one way to view the overlaps between these functions.

![Diagram of overlapping functions of lectures](image)

**Communicating**

Mathematics is impossible without a core of definitions, methods and concepts, which no student can reconstruct unaided. Acquiring these concepts is only the start of the learning process: understanding arises when they are used, through a complex interplay between formal and informal understanding [10]. But the seeds for this process must come from somewhere: they must be communicated. Similarly, although by the end of a course, students should appreciate the topic’s structure and its relation to other topics, this does not come easily.

A teacher can see and communicate the structure and connections, providing a scaffolding for later, more independent development.

The term ‘communicate’ (rather than ‘transmit’) is important. All face-to-face communication involves an exchange of information: at the very least, the audience’s body language indicates whether they understand what is going on. Sensing and reacting to this unconscious feedback is a large part of the lecturer’s job, and the feedback can be made considerably more explicit by employing classroom response systems or setting other in-class exercises [9; 11]. Because the pace and even the content of a lecture can be varied in response to feedback, it has a crucial advantage over written or recorded resources — although these can be used to complement lectures. (The complex issues surrounding handouts and note-taking have been discussed by, among others, [6; 9].)

Another strength of lectures is that they allow communication through multiple channels: writing, visual aids and speech. There is much evidence that working memory is limited, so students are overloaded if information is delivered too fast. However, there is also evidence that auditory and visual stimuli are processed as distinct streams with disjoint memory allocations ([12] and references therein): hence, by using both streams carefully, it may be possible to get more information through the ‘cognitive bottleneck’ of working memory. This ‘modality effect’ gives lecturing an important advantage over methods such as books or podcasts that employ a single channel. It has also been argued [13] that a key purpose of lectures is to make connections between multiple representations of mathematical objects (e.g. algebra and diagrams). Certainly, a lecturer who constructs a diagram or a proof can often communicate its structure better than a book which provides only a finished product and an apparently post hoc justification. (This is an example of “didactical inversion” [14].) Deploying multiple modes and representations also helps to address students’ wide range of implicit assumptions and preferred approaches to learning [8; 15].

Of course, all these benefits rely on lecturers to be good communicators, and to plan the course to take account of their and the students’ typical, rather than optimal, condition. It is also essential to remember that ideas that are merely communicated are rapidly lost unless reinforced by active use. This is why a properly synchronised lecture-homework-tutorial cycle matters — but this need for active learning does not obviate the role of lectures.

**Modelling**

When we stand before a class and do anything beyond reading verbatim from our notes, we bring our expertise to bear on the material and so provide a (positive or negative) model of how a mathematician behaves. This is an under-appreciated aspect of what lectures can achieve.
Lectures can provide students with ‘vicarious experience’, infecting them with the lecturer’s enthusiasm and sense of the subject’s importance. There is evidence [17] that this can help mediate a transition from shallower learning based on ‘extrinsic relevance’ (mostly to assessment) to deeper learning based on the ‘intrinsic relevance’ of the subject. This is an affective aspect of Mason’s principle that lectures are where students are ‘invited to enter the lecturer’s world’.

“The effectiveness of lectures in communicating information rests on clarity, flexible delivery which responds instantaneously to feedback from the audience, and the use of multiple communication channels.”

Another affective aspect is that lectures can cultivate a group dynamic. A large class does not respond just as a collection of individuals but at least to some extent as a group with a shared experience: the germ of a “community of learners” [18]. A teacher who projects a strong ethos of enthusiasm, organisation and self-discipline can certainly have a positive influence on a classes willingness to take their work and the subject seriously — and most of us, I suspect, can recall grim examples of the opposite. Bruce Charlton has argued [19] that the lecture format, by focussing attention on the lecturer, gives them a social status which aids learning. Regardless of the cognitive aspects of Charlton’s argument, this artificial status can certainly be a useful classroom management tool when facing a hundred restless teenagers on a Thursday morning.

Motivation and attitude are elusive qualities, and what works for one class or individual may fail for another: no teaching method should rely on them for its justification and no teaching strategy should rely on one method to provide them. Nevertheless, lectures offer opportunities that other methods do not to influence students’ ethos and their sense of the relevance of a subject, and it seems irresponsible not to make use of this.

**Summary**

Lectures can be, and in practice often are, where students’ learning starts. Because they combine face-to-face interaction with a relatively structured setting, they provide opportunities to address both cognitive and affective aspects of learning, even when students behave not as idealised actors but as the human beings they are.

The effectiveness of lectures in communicating information rests on clarity, flexible delivery which responds instantaneously to feedback from the audience, and the
use of multiple communication channels. Lectures may be complemented, but are not effectively replaced, by written or recorded forms of delivery.

The effectiveness of lectures in *modelling* mathematical reasoning keys into the ‘worked example effect’, and relies on the lecturer’s willingness to narrate strategies and to acknowledge heuristic approaches. The essential message for students to receive is that mathematics involves active processes rather than passive knowledge, and that this applies equally to experts and to novices.

The effectiveness of lectures in *motivating* students to deeper learning involves both communicating enthusiasm and cultivating a ‘learning community’ within the class. A genuine commitment to teaching and an awareness of group dynamics provide a lecturer with significant opportunities to influence how students learn both inside and beyond the classroom.

In each of these functions, the lectures will only be of lasting benefit if they are consolidated and extended through individual and group activities and regular feedback. Traditionally these have been supplied through a cycle of homework and tutorials roughly synchronised with the lectures. Many opportunities may exist to supplement or replace these traditional techniques, but it seems likely that, unless human nature changes or there is an abrupt shift in the resourcing or purposes of higher education, the lecture will remain a popular and effective tool for enabling students to learn mathematics.

**References**