Challenge and support in undergraduate mathematics for engineers in a GeoGebra medium

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Background
This article focuses on a course in mathematics for first year undergraduate materials engineering students of which I have been a teacher now in two successive years. Students in the materials engineering programme have a wide range of mathematical qualifications and experience ranging from A* at A level, achieved just prior to entering university, to GCSE mathematics, achieved several years earlier with no mathematical study since then. The cohort this year includes 72 students who are taught together over two semesters in two lectures and 1 tutorial per week over about 27 weeks. They are assessed through 8 computer-based tests (40%) and a final examination (60%). I was responsible for all the teaching in the first semester (15 weeks).

This is the first year recently in which the whole cohort is taught together and separately from other engineering students. Last year, and several years before this, the students with less experience or lower qualifications were taught separately from the rest who were taught mathematics together with students in electrical engineering. I was responsible, last year, for the students with lesser experience or lower qualifications, a small group of just 16 students, and a reduced curriculum (including basic algebra, trigonometry, functions, calculus and statistics). I engaged in practitioner research as I taught this group, in order to discern, reflect on and analyse the issues that arose in relation to the approach used [5]. The main findings from this research concerned my use of investigative tasks¹ to promote mathematical thinking and its relation to the mainstream teaching of course topics. Briefly, students engaged well with these tasks and showed evidence of enjoying them and engaging seriously with the mathematics. However, these same students struggled with course topics, especially with basic algebra, and when we came to exponential and logarithmic functions demands of the topic took up all our time at the expense of investigative tasks. Thus, a central issue for me as a teacher is how to design a course to integrate a focus on mainstream topics that enables students to develop mathematical fluency and an emphasis on mathematical thinking that encourages conceptual understanding. Before introducing my approach this year, in which I have used GeoGebra, I will say a little about the theoretical principles behind these aims.

Investigation and inquiry in teaching mathematics

Investigational tasks are designed to engage students in mathematics through problem solving in relation to some context, either mathematical or real world. Fundamentally, they encourage inquiry in using mathematics in relation to the problem situation [3]. The principle is that students, in engaging with the problem
situation, will use mathematics in meaningful ways and develop a better understanding of mathematics, both specifically and relationally. Usually, such tasks are used in a collaborative group setting in which dialogue between students and with a teacher is encouraged. Through talking about the problem with others, students develop their own thinking; their participation in negotiative dialogue leads to development of mathematical and problem solving discourses which facilitate understanding [6, 7, 10].

We can see these ideas from either constructivist or sociocultural theoretical perspectives. Briefly, in constructivist terms, in the problem context and setting learners are challenged cognitively to draw on their existing mathematical structures and, through inquiry with others, to reinforce or change these structures (Piaget’s assimilation and accommodation modes, e.g., [9]). In interaction with students, the teacher can offer support or challenge whichever seems to be needed at a given time. The discursive nature of the setting allows the teacher to gain insights into students’ conceptions to enable appropriate degrees of support or challenge. In sociocultural terms, we can see collaborative problem solving in the classroom to offer a mathematical environment for students’ participation. The teachers’ encouragement of modes of participation involving inquiry helps participatory norms to develop through which students engage collaboratively with mathematics in meaningful ways in the problem context [1].

Whichever of these perspectives is taken, key elements involve

- some task that offers potential for inquiry, often involving problem solving [12];
- engagement in inquiry processes with mathematics – asking questions, seeking solutions, analysing outcomes etc [8]; and,
- a collaborative setting in which students can develop mathematical discourse in a relevant context together and with the teacher [6].

The teacher is crucial to the collaborative inquiry setting. The teacher designs the task and the pedagogical processes in which the task is used. The teacher interacts with students to support and challenge as part of the problem-solving discourse. The teacher is influential in creating (with the students) norms of activity and participation through which negotiative inquiry can become an established mode of engagement with mathematics. The references given here refer mainly to mathematics teaching in school classrooms, and therefore address a rather different environment to that in a university. This paper addresses how the perspectives outlined above may translate into the university context.

The activity of a teacher described above can be seen in terms of the teaching triad [3], involving management of learning, sensitivity to students and mathematical challenge. Briefly, management of learning is about designing the context and setting, including the task, to address topics and processes in mathematics and about managing the learning environment to foster learner participation; sensitivity to students is about being aware of the affective and cognitive dispositions and needs of the students so that a suitable setting can be created and challenge can be judged appropriately; mathematical challenge is about offering questions and suggesting lines of inquiry that can be appropriate to learners’ dispositions and needs in the mathematical process to create mathematical engagement and understanding. It can perhaps be seen that the teachers’ judgment regarding appropriate levels of sensitivity and challenge is crucial to the success of the teaching-learning process. When challenge is judged effectively with respect to students’ dispositions and needs, sensitivity and challenge are seen to be in harmony [11]. Thus design of teaching to achieve harmony is an important pedagogic goal. Here I am interested in how such harmony might be achieved in a university setting.

**Context and setting in this year’s module**

There is a considerable difference between planning to teach 16 students of similar mathematical experience and planning to teach 72 with very variable experience as described above. As well as the wide range of experience, the context this academic year included a wider-ranging mathematical syllabus (including topics of matrices, complex numbers and differential equations) and three meetings per week – two lectures and one tutorial. As a fundamental resource for engineering mathematics courses, a set of booklets called the HELM materials (Helping Engineers Learn Mathematics) have been developed in a major project over several years (http://helm.lboro.ac.uk/). These offer a text-book style exposition of the mathematical topics, with examples and exercises. The relevant booklets are provided free for each student and offer a basis for self-study in these topics. As well as these elements of context, I work in a university that prides itself on being student-friendly and maintains two Mathematics Learning Support Centres (MLSC) in which one-one mathematical support is available daily to any student who seeks help at whatever level (http://mlsc.lboro.ac.uk/).

Thus, the context of the course includes various kinds of student support which can be seen to contribute towards a sensitivity to students’ needs. Nevertheless, the context also includes the norms and expectations of university teaching including lectures with large cohorts of students in fixed seating environments, with rather more flexibility on tutorial settings. Design of teaching takes place within this context and the elements of management of learning (ML), sensitivity to students (SS) and mathematical challenge (MC) are conditioned by the context. Design determines the setting within a philosophy of learning and teaching.

I see myself as a very experienced mathematics teacher with many years as a reflective practitioner with experience as practitioner-researcher and researcher of practitioner research, mainly at school level [3, 4]. I am a novice where
university teaching is concerned, especially with lecturing and the use of lectures as an effective teaching-learning tool. With the 16 students last year, I worked mainly in classroom mode and got to know well those students who attended. With 72 students, classroom mode would not be possible, and so I needed to think in terms of lectures and tutorials and a metaphor of ‘delivery’ of the curriculum. Nevertheless, I wanted to try to encourage mathematical thinking beyond the routine, and students’ engagement with mathematical challenge through inquiry and investigational tasks. This was a challenge for me, and I decided to use GeoGebra to assist me in achieving these goals. Each week, students would be offered two lectures in which I would present mathematical ideas, concepts and examples, and one tutorial in which students would be offered tasks to work on using GeoGebra. For the tutorials I requested a computer laboratory. This explains the setting for the course.

Design of teaching for learning using GeoGebra

The material to be covered was organised over the 15 weeks. For the lectures, I chose to complement the HELM materials rather than using them as a basis for lectures. Thus, I planned PowerPoint presentations to represent the main ideas, using layout and diagrammatic forms to emphasise ideas and concepts. I used the OHP to offer examples and in-class exercises. Associated with this form of presentation, I prepared a tutorial sheet of tasks related to the lecture material. These tasks were designed to be tackled in a GeoGebra environment in a computer laboratory, and to be continued in the students’ own time using GeoGebra. Within the laboratory, students were seated one to a machine, and encouraged to talk with each other as they worked on the tasks.

I thought hard about the design of presentations and tasks. Starting from an analysis of the mathematical concepts that students were intended to address and understand, I tried to offer opportunity for students to engage with concepts meaningfully. I had come to know GeoGebra as part of a programme for foundation year students (see [2]) as part of which we prepared tasks to engage students in mathematics. I appreciated particularly the dual nature of the screen, the possibility to see algebra and geometry side by side, and the alternative possibilities of inputting a function either algebraically or geometrically. The early weeks of the module focused on functions and equations and so I planned tasks to encourage students to engage with functions conceptually through seeing how the functions appeared when represented geometrically.

For example, a task focusing on inverse functions read as in Fig 1. A range of functions was given, starting from simple linear functions, through quadratics and cubics, to trigonometric, exponential and logarithmic functions. Students were encouraged to determine their own level of difficulty in deciding where to start and which functions to focus on.

Taking the domain as the entire set of real numbers \( \mathbb{R} \) (the whole of the \( x\)-axis) draw graphs of the following function rules in GeoGebra. Look at horizontal and vertical lines crossing the graphs and decide which of the graphs shows a function AND which of the functions has an inverse.

For the functions which do not have an inverse, explain why not and how you might restrict the domain of the function so that an inverse is possible.

Fig 1 – A task on inverse functions

In preparation for this task, the lecture had included a demonstration using GeoGebra of a variety functions with inspection of vertical and horizontal lines crossing each graph and questions about the nature of one-one and many-one functions. It had gone on to consider the meaning of inverse, the relation between a function and its inverse, and restriction of the domain on one-many functions to make an inverse possible. The image in Fig 2 was one of those drawn dynamically in GeoGebra. This image, showing reflection of \( y=x^2 \) in the line \( y=x \) was compared with the graph of \( y=\pm\sqrt{x} \), as shown in Fig 3.

Fig 2 – Demonstration of inverting a function by reflection in \( y=x \); need to restrict the domain

During the lecture, I used both pre-drawn images captured in PowerPoint and dynamically drawn images created on the spot in GeoGebra to explain and illustrate the concepts of function and inverse. During the tutorial, students were meant to experiment and explore, and to discuss their work with their colleagues and with myself and a graduate student. Thus, my intention was that students would start to appreciate a relationship between the nature of a function, the various representations and the subtleties of different kinds of functions and their inverses.

When we came to solving equations I wanted students to appreciate links between functions and equations, and to see how their solution of an equation was related to the functions of which the equation was composed. The task
shown in Fig 4 was designed for this purpose -- the equations listed are a representative subset of those offered to students.

Consider the following equations and associated functions:
First solve each equation algebraically according to the usual methods.
Then draw the graphs of the functions on the right.
Find points of intersection of the pairs of graphs and look to see how these relate to the solutions of your equations.
Compare your answers with those of a colleague.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 3x + 4 = 7</td>
<td>y = 3x + 4, y = 7</td>
</tr>
<tr>
<td>2. y - 3x = 5 + 4x</td>
<td>y = 2 - 3x, y = 5 + 4x</td>
</tr>
<tr>
<td>3. ( \frac{5x}{2} - 7 = 0 )</td>
<td>y = 5x/2 - 7, y = 0</td>
</tr>
<tr>
<td>4. ( 2x^2 - 5x + 6 = 0 )</td>
<td>y = x^2 - 5x + 6, y = 0</td>
</tr>
<tr>
<td>5. ( x^2 = 8 )</td>
<td>y = x^2, y = -8</td>
</tr>
<tr>
<td>6. ( x^2 = 3x^2 )</td>
<td>y = x^2, y = 3x</td>
</tr>
<tr>
<td>7. ( x^2 + 3x^2 = 0 )</td>
<td>y = x^2 + 3x^2, y = 0</td>
</tr>
<tr>
<td>8. ( \frac{3}{x} = \frac{-1}{5x} )</td>
<td>y = ( \frac{3}{x} ), y = ( \frac{-1}{5x} )</td>
</tr>
</tbody>
</table>

Create some of your own equations and associated functions.

A subsequent task asked students to multiply complex numbers in polar form and to compare the graphs of the original functions with the graph of the product in order to emphasise relationships between moduli and arguments. Conversely, they were challenged to take a complex number, graphically, as starting point and show it as a product of two other numbers, checking by calculation. Thus, tasks were created to address central concepts in exploratory ways using the GeoGebra medium. It was an aim that students would relate their observations in GeoGebra to the concepts introduced in lectures and reinforced in the HELM materials.

Outcomes and reflections
Students who attended tutorials worked on the tasks individually using GeoGebra. Some dialogue between students was evident, but it seemed to focus more on technical concerns, for example with the use of menus, than in engagement with concepts. As I moved around the lab talking with students, I tried to open up conversations focusing on the concepts the tasks were designed to address. This was hard going. It was clear that many students became quickly technically competent with GeoGebra and able to input and draw graphs as the tasks required. They asked questions confidently about the use of the software, and these questions were largely about how to achieve results in GeoGebra. There were few questions about the mathematics. When I intervened in their activity to ask questions about mathematics, responses were monosyllabic. If I drew two students together to discuss a concept, I found myself doing most of the talking. I recognised a number of factors here for the students: unfamiliarity with a mathematical discourse, unwelcome interruption of key pressing and image creation, uncertainty (and possibly apprehensiveness) about what I was asking and whether they could respond. When there was no clear response to probing questions, it seemed better to point out important aspects of the mathematical concepts, than to risk there being little or no attention to the concepts. Sometimes, this generated a dialogue, but often students waited politely until I had finished and then went back to their activity.

In lectures, I was able to emphasise the important concepts through demonstration and example, with some in-class exercises during which I circulated as far as a lecture theatre would allow. Although attendance was variable, the number of students generally made small group activity difficult, as did students’ expectations of what a lecture would or could involve. Students seemed content to watch, listen and write notes, less content to engage in discussion of ideas. As I circulated, I could see that most students worked on the set task, but, as in tutorials, it was hard to generate a mathematical discussion. Early in the module, when we focused on partial fractions, I offered a demonstration at the OHP of several approaches to creating
the fractions in what I saw as being largely a technique-bashing session. In order to finish, I ran a little over time. As students rushed to leave, more than half of them, as they passed me on the way out, said “thank you”! I was rarely thanked in this way and never for my attempts to use GeoGebra to foster understanding.

In terms of the teaching triad, my management of learning included the preparation of lectures and tutorials, design of tasks and attempts to provide for students’ understanding. Use of GeoGebra formed a significant strategy to foster conceptual understanding. Here the tasks were designed to encourage exploratory activity, including questions to stimulate inquiry, and collaboration involving discussion between students. On reflection, I could have used lectures to set up questioning and inquiry, to introduce investigative tasks and generate discussion. This will be part of my agenda next year.

Sensitivity to students can be seen mainly in the way I have tried to address material to a wide range of experience, offering tasks at different levels with access for the least experienced, and with questions to challenge the most experienced. Support was provided in several ways: the free HELM materials with back-up notes and exercises; a special session each week (with another teacher) for those who were struggling (but few attended) and one-one help in the MLSC. The numbers of students, variability in attendance and constraints of fixed seating lecture theatres and the computer laboratory militated against getting to know students as individuals and of getting a sense of student needs in any but the most general way. Mathematical challenge was mostly in the tasks set in GeoGebra sessions. These attempts to engage students in mathematical exploration and encourage conceptual understanding but with limitations as described above. I feel overall no sense of achievement of harmony.

At the end of the first semester, students were asked to complete a university-produced evaluation sheet (not specific to the module) which generated many positive remarks, but there were also specific reservations expressed. For example, a small number of students did not like PowerPoint presentations and three students said that GeoGebra was “a waste of time”. In order to get clearer feedback on students’ experiences over the two semesters, we are planning a focus group meeting of selected students to generate discussion about their experiences and preferences, and to gain insight into learning modes and levels of understanding. In addition, we are talking with colleagues from the Materials Engineering department about generating problems related to engineering practice.

In preparing for next year’s planning, I am reluctant to abandon my aims for GeoGebra as a tool directed at generating conceptual understanding through exploration and inquiry. These reflections point to a need for developing an alternative culture in which students are clearer participants in conceptual inquiry in mathematics and in which an inquiry discourse is generated. Having seen the process through once, I am eager to talk to others who use GeoGebra in similar contexts, who perhaps have ideas as to approaches that could achieve these aims.

References


