The Skills needed by Postgraduates who teach Mathematics and Statistics

In October of 2005 the Maths, Stats & OR Network ran a series of workshops for postgraduates who teach Mathematics and Statistics, initiated by Michael Grove [1]. For an individual postgraduate’s experiences of teaching, see [2]. And for a novel and important role for postgraduates, see [3]. The postgraduate workshops represent a useful addition to the CPD activities that the MSOR Subject Centre is building up [4]. As the workshops progressed a unifying pattern began to emerge of the training needs for postgraduates and we develop this further in this article.

The workshops covered:

- Planning and Delivering Small Group Teaching
- Facilitating Problem Solving Classes
- Encouraging Participation: Motivating and Sustaining Student Interest
- Assessing Student Work and Providing Feedback

An interactive session concluded each workshop giving participants the opportunity to discuss their individual concerns and share advice. Ben Cosh, Bill Cox, Joe Kyle, Duncan Lawson and Chris Sangwin, all experienced HE lecturers in mathematics, contributed sessions to the workshops, which were run at the University of Birmingham, University of Bath (Swindon Campus), and the University of Central London. A set of draft notes was issued for each session, which may be accessed at:

http://mathstore.ac.uk/workshops/postgradwhoteach2005/index.shtml

These materials will evolve in response to feedback. There was a good spread of experience across the participants which ensured a lively and productive exchange of ideas and views.

A survey of postgraduates’ training needs [1] confirmed that most are involved in running seminars, tutorials or problem classes of some kind in mathematics or statistics, 40% are involved in marking exams and about half marked coursework. Postgraduates perform such a wide range of duties directly interacting with students, that it was not initially clear how best to support them. However, during the course of the workshops a pattern emerged, with the postgraduates’ help, which seems to provide a natural approach to such training. This is described here, but of course is likely to evolve in response to further feedback. In any event, our main objective was to ensure that all participants had the opportunity to receive targeted advice.

Normally postgraduates are teaching in a small group environment, or are directly involved with either demonstrating how to solve problems, or assisting students in their attempts to solve problems. As far as assessment is concerned, they are normally marking formative student assessment and providing feedback. So, while they are not usually responsible for a whole course, postgraduates are actually in the front line when it comes to supporting student learning. We can perhaps summarize the basic skills needed by the postgraduates as follows. Postgraduates have to:

- ENTHUSE the students about mathematics
- ENGAGE the students in productive mathematical work
- EXPLAIN mathematics to students with varied backgrounds
- EVALUATE student work and mark fairly
While the workshops were not structured in this way, this method evolved naturally as a way to focus on specific skills postgraduates needed in the context of mathematics and statistics. These skills are central to teaching mathematics by any means, for anyone, and they are difficult to master, particularly for those new to teaching.

**Enthusing Students about Mathematics**

The postgraduates were encouraged to at least give the appearance of enthusiasm, even if they found a topic uninspiring. In fact, with sufficient imagination, there isn’t much mathematics that can’t be made interesting. Self awareness and self discipline are needed to adapt when you feel you are becoming jaded or boring. A teacher might raise enthusiasm by asking such questions as:

- History of the topic, where did it come from and why?
- Where does it lead to, what areas of mathematics/statistics are built on it?
- What sort of applications does it have?
- Can I derive any of the results by different methods?
- Can it be generalized?
- How is it related to my own particular area of interest?
- Why do the students need this, and why should they find it interesting?
- What concepts are at the core of the topic, and where else do these occur – how did the original developers of the ideas find their way through them?

Students are more likely to become enthusiastic about a topic if they appreciate the key points upon which it is based. These should be emphasized to the students so that they have anchors for their ideas, and motivate and explain why these points are so important, highlighting any interesting features. Provide examples or asides that may be of direct relevance/interest to them (which does not necessarily mean in their own subject).

The students’ curiosity can be aroused by posing ‘interesting questions’ – and persisting in getting answers. An occasional short amusing story related to the topic in hand can sometimes spark interest. Sometimes, you can ask for their help, for example on something they may have covered in another topic. In talking to the students, try to justify sensibly, as well as logically. Discuss their notes with them. Choose problems that are ‘just the right level’ – too easy can be boring, too hard demoralising. If all else fails in eliciting interest, then regular reference to the examination usually does the trick.

As a group exercise the postgraduates were asked to think of a tedious piece of mathematics and then treat it in an interesting way. You will be glad to hear of one workshop at which they couldn’t think of anything boring in mathematics – they loved it all! One group suggested the topic of convex sets – what could be more boring than the definition of a convex set? But, why do we need convex sets and what makes them so important and actually very interesting? Well, in a convex set you can always move between any two points by the simplest path imaginable, a straight line – that sounds quite interesting, surely? Vertices representing optimal points can therefore be located by scanning a convex set with straight lines, again, surely interesting? But how many sets do we know are convex – aren’t they just a rather unrepresentative case and therefore not that interesting really? Ah, but we can always break up most sets into separate pieces that are convex, treat those separately then put them back together again – in other words, what we can do with a convex set you can usually stretch to any set. The point is that while the dry definition of a convex set may be pretty uninspiring, the teacher can make it much more interesting with a few minutes background such as that outlined above.

**Engaging Students in Productive Mathematical Work**

Postgraduates are often employed in a tutorial or problem solving situation, and one of their main tasks is to actually get the students involved, working productively and keeping them at it. The importance of preparation in building in activity was emphasized. The purpose of the session needs to be made clear, with clear instructions for any activities. Learn as much as possible about the students and remember that they will not necessarily be as motivated or able as yourself.

To set the right atmosphere start the session off in a business-like way. Ensure that they all have the resources necessary for the session. Indicate some sort of schedule, and set clear ground rules about orderly conduct of the class. Once started, keep things going. Be everywhere and make sure everyone is active. Help them with entry methods to problems. Tell them to try anything at all, and that sensible guessing is fine. Keep to schedule and ensure even coverage of the material. Encourage students to use their notes, books, etc. and to create their own learning materials. When working through problems on the board, you are simply the scribe – you should tease most of your working out of the students – and not just the good ones. As an exercise in
engaging students, one workshop had a role-playing session with the postgraduates playing different stereotypes in a small group environment. The object was to get all students engaged in the activity, even though some had particularly recalcitrant roles! This worked very well.

If a student is stuck and not getting anywhere, don’t rush to rescue them completely, but just give them a little help along the way. For example in solving a differential equation by the integrating factor method many students have difficulty even starting. Don’t jump straight in and find the integrating factor for them. Instead, give them a few suggestive products to differentiate, get them to think about the purpose of the integrating factor. For the very weak student, tell them to find an example in their notes and repeat it line for line with the new equation. The overall purpose is to get the student working and thinking, not necessarily to show them how to solve the equation.

Related to the job of keeping the students engaged is the difficult task of maintaining a disciplined and productive working atmosphere. This was raised as an important issue for postgraduates and is sometimes a problem even for experienced staff. As they may have only recently graduated from being students themselves, postgraduates may lack confidence in their role and status. On the other hand they have the advantage that they are ‘closer’ to the students and may appreciate their difficulties more readily. To help keep things under control, the postgraduates were encouraged to be clear about their duties, responsibilities and status. Set ground rules early on and stick to them. Keep order. Never be rude, sarcastic or derogatory. Allow leave only for essential purposes. Keep on task. Be aware of any diversity issues (e.g. non-native speakers).

Explaining Mathematics to Students with Varied Backgrounds

Explaining is of course one of the key arts of teaching [5]. You don’t explain by telling, but by listening, learning, and dialogue with the student(s). You first have to find out what the students know so you can ‘get on their wavelength’. And be sure you know what ‘know’ means for your students. Ask the typical first year if they ‘know’ the product rule in differentiation and they will invariably say yes. But whether they will readily recognise its reversal in the integrating factor method mentioned earlier is far less certain. Having established a common language and starting point lead the student through gradually, dangling the next step just close enough to encourage them to move forward. You may have to break off and tell them to go and think about it for a while. Don’t be afraid to let them go up some blind alleys – teach them how to backtrack and start again. And when they finally get it be pleased with them and hurry them on to the next topic. Never use any sort of negative, derogatory or demeaning response to a student’s question. Be polite and helpful, but you don’t necessarily have to give the student everything they ask for.

The postgraduates were urged to prepare well for their sessions, so that they can do all the problems in the way that the students are supposed to do them. With their superior knowledge they may know a number of different methods for tackling a given problem, but how is the student supposed to do it? For example an expert may do a partial fractions decomposition very quickly by the cover up rule, but have the students been introduced to this? On the other hand, if the student is on top of the subject, then they might be exposed to the other methods to stretch them a little. Postgraduates were therefore advised to develop a ‘global’ overview of a topic, with a range of different approaches, so they can adapt to different situations as required.

As a group exercise, the delegates were asked to choose a particularly difficult piece of mathematics or statistics, at any level and prepare a short presentation (5-10 minutes) explaining this topic to a typical first year student in terms they are likely to appreciate. At one workshop a postgraduate proposed a presentation on homology groups, and I began to get worried that the exercise might misfire. But in fact, he made an excellent job of it, and demonstrated perfectly how even the most difficult ideas can be explained simply by: paraphrasing technical terminology, notation, etc; careful use of pictures; appropriate analogues; providing a range of viewpoints, converging and diverging. At a more mundane, but no less challenging level, another workshop looked at explaining completing the square to engineers. In this case the explanation is usually made difficult because students are not given a clear view of the key ideas involved (expansion of $(a + b)^2$ and $A - A = 0$) and lack sufficient skills and fluency in the use of these, therefore, another key pre-requisite of good explanation is ensuring that the basic key components are fully understood beforehand.

Evaluating Student Work and Marking Fairly

The participants also had the opportunity to mark authentic student work. They marked three students’ attempts at expanding $(2 + 3x)^6$ by the binomial theorem and were asked to discuss what a marking scheme might look like. The table below shows the spread of marks awarded at one workshop (out of 6) for the three
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Bill Cox

Attempts. In the actual exam, all students made different types of errors but obtained the same overall mark of 4/6.

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The important point emphasized here is that one has to be very clear about the skills being assessed – the learning outcomes being examined. For example, only one student made the common mistake of omitting the 3 in the expansion, and so got all but one of the coefficients wrong. But this cannot be penalized too strongly because he had remembered the binomial theorem formula, evaluated the binomial coefficients correctly, got the correct powers, etc, and yet appeared to get ‘everything’ wrong because of one slip at the beginning. Some postgraduates thought lack of explanation should be penalized heavily, but there was no hint in the question that this was called for. While discussing such issues the postgraduates came to see the importance of a good marking scheme. If nothing else, this exercise taught them that there was plenty of room for debate about marking even the simplest questions, and indeed in their feedback, a number of them said they would mark more thoughtfully in future.

Some Lessons Learned

Formal question and answer sessions were introduced in the second and third workshops, with postgraduates submitting beforehand their most pressing questions. This provided a lot of material for discussion. The questions and suggested responses (on issues of small group teaching and running problem classes) from the second workshop were written up and distributed to all delegates and can be found on the Network’s web-site (http://mathstore.ac.uk/workshops/postgradswhteach05/index.shtml). They are almost certainly questions many postgraduates will have considered at some time or other.

In their feedback, the delegates gave clear directions for the sort of training they value:

- Lots of practical and hands on exercises
- Opportunity for sharing and discussion with others
- Nitty-gritty no-nonsense explanation and advice
- Getting their individual questions answered
- Unified overview of issues
- Emphasis on the importance of teaching
- Enthusiastic and dynamic delivery

We believe we have made a solid foundation on which to build this sort of provision. Also, there were some suggestions for other sessions that they would like to see – especially ones covering a wider range of topics and appropriate to non-native speakers.

References