Introduction

A vital factor in the accessibility of mathematics courses and in the quality of graduates produced is the nature of support given. In this paper we describe briefly aspects of the support offered to students at Sheffield Hallam University and, particularly, the part played in this by technology and mathematical diversity.

Personal support in the form of tutorials, drop-in sessions and one to one meetings forms a key element in fostering student development. Peelo and Whitehead [1] found that approximately 75% of students in a survey cited their lecturer or personal tutor as the first port of call when they encountered problems. An overall aim, however, is that students will become more self-sufficient as their course progresses. Towards the end of their course, the best students require little detailed support, and will interact with staff more in terms of general direction and discussion of points of interest. In our experience, the development process is aided when students explore a variety of ways of tackling the same problem. This variety comes from the use of different technological tools, and also by looking at a problem mathematically from alternative points of view. As noted by Cox [2], interest can be sustained by carefully chosen examples building on fluency with basic principles.

Preparatory and first years

A small number of students enter the mathematics degree at Sheffield Hallam University by first taking a preparatory year in mathematics. These students often require substantial support when they first join, and this is catered for by extra tutorial time and small group work. The use of calculator and computer technology, including Excel and Derive, is introduced during this year and students enter the first year of the degree with a somewhat greater level of exposure to technology than a typical A-level student.

Many students entering the first year of the degree will also require substantial support. In the first semester, students are assigned to academic tutorial groups of about six students. Each group undertakes a project and meets with their academic tutor weekly. Academic tutor group meetings allow academic tutors to informally monitor students’ general progress during the first few weeks and also lay the foundation for further group work. It is not unusual for students who started in the same academic tutor group to still choose to undertake group work together and offer mutual support in the final year.
When technology is introduced, its use is not intuitive and a significant amount of detailed support is needed to help students feel confident with it. The advantage of technology at this stage, however, is that it provides a rapid means of verifying answers. Checking that a function’s zeroes match up with a graph produced on a calculator or in Excel, or that a derivative worked out by hand matches up with an answer from TI Interactive or Derive becomes the natural thing to do. The beginnings of a scaffold for self-support arise from an ability to clearly analyse discrepancies between answers obtained in different ways.

As well as gaining confidence with the “nuts and bolts” of finding solutions, an important facet of student development is the ability to question answers from the point of view of plausibility, compared to the real situation they are describing. As an example of how the use of technology can make this process richer, the question below is from an introductory first year assignment. File 1 contains population data and file 2 contains data from the recording of a musical note.

Example question

(a) Consider the four models:

- **Linear model** \( y = mx + c \)
- **Exponential model** \( y = ae^{bx} \) or \( y = a + be^{cx} \)
- **Quadratic model** \( y = ax^2 + bx + c \)
- **Trigonometric model** \( y = a \sin(bx + c) \)

Work out which is the most appropriate model for each of these cases below, and fit it:

(i) The data in file 1. Use your model to predict what population would be in the year 2000. Can you find out what it actually was?

(ii) The data in file 2.

(b) (i) For data set (i) write a brief explanation of why your choice of model is appropriate, and whether and why your projection may be sensible or not, which is suitable for your virtual boss at your virtual work to read.

(ii) For data set (ii) write a brief explanation of why your choice of model is appropriate, which is suitable for your Course Leader at University to read.

(iii) For one of your two data sets write a brief explanation of why your choice of model is appropriate, which is suitable for an un-mathematical student friend to read.

In addition to the mathematical thought required, this question embodies the use of communication skills in justifying the choice of model to different audiences, namely a non-mathematical professional, a specialist mathematician and a non-mathematical friend.

A typical student at the end of the first year will have reasonable facility with maths techniques, and will not be afraid to use technology as part of a mathematical investigation.

Second year

In support terms, an important aim in the second year is to shift the emphasis away from detailed “line by line” help. Students are beginning to build up a battery of techniques and hence problems can be examined from different starting points. This allows a check on the veracity of solutions and also an assessment of the sort of information provided by each method. As an example, early in the second year, students construct an Excel based stochastic simulation of carbon 14 decay. The comparison of the ensuing graph with the usual exponential solution acts as a mutual check on both methods, and also points up the different information coming from the two solutions. As shown in Fig 1, the random nature of radioactive decay is not evident from the exponential function, but the exponential function does give a good description of the long term trend in activity. Conversely, a stochastic simulation is inherently random and it is necessary to repeat results and take averages before deciding whether the two methods agree. Although some students will still need help with Excel details, tutorial support is aimed at encouraging students to question the nature of, and difference between, solutions.

The theme of developing confidence and self-support by comparing approaches is continued through the year. The comparison of an analytical and Runge-Kutta solution to an ordinary differential equation (ode) provides a check on each. If solutions do not agree then students broaden their understanding by asking questions such as:

- Is the difference in solutions within the bounds of typical numerical error?
- Is the numerical solution unstable?
- Which of the two solutions might give a solution which better describes the facts?
Phase plane plots give much information about the behaviour of the solution of a system of odes which can then be cross-checked with time-dependent solutions. Problems with well known solutions, for example maximising the area enclosed by a given perimeter, act as a testing ground for genetic algorithms. The correctness of a Fourier coefficient calculation can be verified by a plot gradually building up the series in the presence of the function being approximated.

An example of where students can use their experience to choose from a repertoire of techniques is provided by a case study on selling drinks cans which they undertake towards the end of the year. Student groups decide how to price their cans each day by considering prevailing weather conditions and the likely pricing strategies of other groups. This process should be informed by data they accumulate as the study proceeds. It is left to each group to decide how to extract information from the data by, for example, using empirical fits or constructing tables of possible profits.

Once a demand for cans has been allocated to a group then these cans have to be distributed around the network of towns shown in Fig 2 in proportion to their populations. This leads to consideration of, for example, the contrasting merits of a genetic algorithm compared to the use of logical functions in Excel. Deciding on the best strategies for pricing and distribution can lead to lively debate both within and between groups. Through the presentations associated with this case study, the students are able to discuss and share their different approaches.

Final year

By the beginning of the final year, students have generally developed good mutual support networks and can largely find their way through difficulties in detail, either in a mathematical argument or in using a new technological application. Projects are supported by weekly meetings with supervisors. These meetings usually focus on general project directions and a review of progress rather than on detailed help. Now is the chance to discuss with the student what might be gained by different mathematical approaches, and the use of different technology. What can be gained for instance in an epidemic model by taking a cellular automata approach in contrast to developing a system of odes or a pde? What are the pros and cons of using a stream function-vorticity approach to a fluids problem rather than working with primitive variables? For the best students, meetings with the supervisor comprise a progress report together with their ideas of the next step. The supervisors function is then one of reassurance and encouragement. Weaker students may still have substantial difficulty with particular methods, and generating ideas for proceeding with the project investigation that they have undertaken. Support is therefore still flexible and on occasions at a “line by line” level.

As noted with the example first year assignment question above, an important part of the mathematical process is the production of a coherent account of methods used and results found. This culminates in the final year project report, but also features in the majority of assignment work throughout the course. While some students have excellent writing skills, perhaps having studied A-level English, others initially have difficulty in writing a coherent argument. This is addressed by including amongst assignment work some short pieces of writing which are dissected in detail, with extensive feedback given to students. An interim report is required for the project, and this provides a final opportunity for feedback, specifically on English style, before the final report is written.

Some student reactions

Through the on-line log kept by students [3] and other means, student reactions to their course and assignment work are readily available. Generally, the purpose of using technology and alternative approaches is recognised by students, and the following is a selection of reactions to their creativity is in the final year project.

Fig 2 – Distribution network for drinks cans case study

"By the beginning of the final year, students have generally developed good mutual support networks and can largely find their way through difficulties in detail, either in a mathematical argument or in using a new technological application.”
the example first year assignment and the radioactivity simulation outlined above

First year assignment

“Firstly, I would just like to comment on how much I have thoroughly enjoyed this assignment, and the outcome of producing models to predict future population.”

“The use of Microsoft Excel in previous assignments has helped to develop and build on skills that I already had, and to prepare us for the mathematical functions and formulas used in creating different models within this assignment. We have learnt about how to implement and experiment with linear, exponential, quadratic and trigonometric models.”

“This assignment is by-far the most ‘hands on’ out of all modules that I am currently taking, and I feel this helps to understand the topic in much greater depth. Carrying out predictions on future populations and commodities, has allowed me to deepen my knowledge of models, and how to use them as a valuable tool to look into trends and what affect they have on the future.”

Radioactivity simulation

“Over the week I completed last weeks tutorial, I got stuck quite a few times but managed to work it though by myself.”

“If I hadn’t been so hungry I could have quite happily stayed in the computer room for hours longer.”

“My simulator is working fine over the week. I had completed some more work on it and it produced a lovely curve shape in the data.”

Conclusion

Personal support by members of staff remains essential to a good and productive student experience. This can be optimised, however, by developing self and peer support among our students. Technology and a variety of mathematical paradigms aid students in checking their work and in asking meaningful questions about the validity of their solutions. Self support is encouraged because students have means of recognising errors and identifying their source. Mutual support is also encouraged as students debate different methods of approach. Finally, student reactions are positive as exemplified by the sample included above.

References

