History and current status

This article briefly describes the scope and some of the underlying pedagogic thinking behind the development of an extensive set of objective tests that are now available for free download. In the early 1990's, my students and I started writing a system called Mathletics\(^1\) using Question Mark Designer, reviewed by Kyle [8] and discussed by Greenhow [6,7]. This comprised some 4500 hard-wired questions with tests that randomly selected from this library. A principal feature was the extensive feedback offered to students. The tests proved extremely popular with 300-400 students taking up to 25,000 tests per year at Brunel University. Despite this popularity and usefulness, this hard-wired system was ultimately a dead end and development stopped in 2000, although very substantial use continued for two more years after that.

With the advent of Question Mark Perception's open coding in versions 2 and 3 (but not 4), it was decided to build on the work of Professor David Hewitt of Monash University, Australia (private communication) who used Javascript to incorporate random parameters into his chemistry questions. This means that one is able to author an entire class of question in one go (we call these question styles) that produces thousands or millions of question realisations seen by the students. The groundwork for using randoms within mathematical displays (using MathML) and diagrams/schematics (using SVG) followed (see [3]).

The current situation is that some 1800 question styles have been authored mostly covering lower level mathematics (GCSE to level 1 university). I am currently in discussions with Question Mark Computing to discuss updating to their latest (secret!) product, completely bypassing Perception 4. This may or may not happen; other systems are waiting in the wings, e.g. Open Mark (see [2]) and more advanced systems that offer symbolic manipulation, e.g. STACK [12] and Maple TA [9] that have a number of advantages for (more advanced) mathematics courses. Having said that, there are substantial advantages to the nuts-and-bolts coding of e.g. Javascript whereby one has complete control over and can author very full feedback. It may well be that Mathletics turns out to be of most use in situations where one does not need the full power of a symbolic engine, for example at A level and for students in departments where mathematics, statistics and numeracy are taught to some extent e.g. ‘quants’ courses in economics, engineering and design, health studies etc. Indeed at Brunel University these departments are increasingly using Mathletics and discussions are underway with our School of Education to export the technology to their PGCE course competency tests in grammar and phonics (sample questions show that random parameters can be exploited effectively here too).
Authoring objective questions involves very clear specification of the question design and interacting technical and coding issues. This makes it a rich source of student projects and some of the Mathletics system has been written by final year and postgraduate students. Having established a critical mass of question styles in some areas (see below), our attention is increasingly turning to evidence-based evaluation of the tests using the many answer files collected over the last few years (see [1, 4, 5]). In particular, the first two papers aim to look at how students make mistakes using consistent but incorrect methods, termed mal-rules, and how to categorise such mal-rules into an over-arching error taxonomy, partly to make student and whole class profiling more manageable. Such considerations are, of course, independent of any particular assessment system.

Scope

For ease of navigation, the download site categorises and bundles the Mathletics contents according to nominal levels, see Table 1. Obviously these are not strict boundaries and much of a typical year one calculus module at university would appear at A level, especially Further Maths A level. It is a rather sad fact that many students at university taking subjects such as Business Studies would find the GCSE tests challenging; perhaps they once knew this material, but having not studied maths for two years, they are no longer able to do even simple percentages. Equally some of the decision maths A level is first encountered in level 2 mathematics modules, albeit at a much more sophisticated level. So the boundaries are very blurred.

The database itself is structured according to mathematical topic/subtopic, questions being tagged according to Table 1. The structure is usually easy at lower levels, but can be open to debate; for example, should a rational function integration question style be placed under integration/rational functions/partial fractions or under algebra/algebraic fractions/partial fractions/irreducible factors/linear & quadratic. The distinction is important because the ‘Related material’ button (see Fig 1) uses the question topic to look up related web sites. Similarly syllabus tags, such as A level C3, link to textbook level.

<table>
<thead>
<tr>
<th>Student group</th>
<th>Contents</th>
</tr>
</thead>
<tbody>
<tr>
<td>GCSE level</td>
<td>Arithmetic, fractions and percentages</td>
</tr>
<tr>
<td>GCSE level</td>
<td>Algebra diagnostic tests, handling brackets &amp; indices and solving simple equations</td>
</tr>
<tr>
<td>A level</td>
<td>Arithmetic, fractions, percentages, complex numbers and number bases other than 10.</td>
</tr>
<tr>
<td>A level</td>
<td>Algebra diagnostic tests, handling brackets, indices algebraic and rational expressions, and solving equations</td>
</tr>
<tr>
<td>A level</td>
<td>Rules of differentiation, techniques of integration, binomial theorem and straight lines.</td>
</tr>
<tr>
<td>A level and beyond</td>
<td>Algorithms and linear programming.</td>
</tr>
<tr>
<td>First Year University</td>
<td>Determinants, matrices and solution of matrix equations.</td>
</tr>
<tr>
<td>First Year University</td>
<td>Vectors, kinematics, dynamics and statics (including trusses and beams).</td>
</tr>
<tr>
<td>Second Year University</td>
<td>Vector calculus, Laplace transforms, Fourier transforms, Fourier series and ordinary differential equations.</td>
</tr>
</tbody>
</table>

Table 1 – An edited version of the contents table for Mathletics downloads
Economics applications – elasticity, growth and decay, input-output analysis, interest, market share, optimisation, percentages (appreciation & depreciation, index numbers, inflation and ‘two changes’) & theory of the firm. (This will be developed over the summer to include unconstrained and constrained optimisation using Lagrange multipliers (see Metal web site [11]).

Functions – exponential, log and trig functions.

Fourier series – applications, basics & complex and half-range.

Fourier transforms – basics & properties.

Integration – algebraic functions, by parts (exponential, inverse trig, logs, trig), by substitution (to be developed soon), polynomials, rational functions & trig functions.

Laplace transforms – applications to modelling, applications to ODEs, direct, inverse & properties.

Matrices – addition, determinants, eigenproblems, general, inverse, LU factorisation, multiplication, non-square systems, norms, norms of vectors, row echelon & trace.

Mechanics – dynamics, kinematics, statics & vectors (this covers all of M1 at A level and some of M2).

Numbers – BODMAS, complex, fractions, modular arithmetic, number bases, percentages, scientific notation & surds.

Numerical methods – Newton-Raphson.

Ordinary differential equations – first-order (Bernoulli, exact, homogeneous, linear, separable), formation, higher-order (application, direct integration, homogeneous, identification, inhomogeneous), identification & modelling.

Probability – addition rule, Bayes theorem, definitions, discrete random variables & multiplication rule (to be mounted).

Statistics – correlation, descriptive stats, distributions, forecasting and many tests (to be mounted).

Vectors – general, scalar products, triple products, vector calculus (curl, div, grad, Laplacian, line integrals, scalar potential, surface integrals) & vectors products.

Some pedagogic considerations

Assessing mathematics has specific problems in communication with any system; Mathletics has largely sidestepped the input problem by focussing on multi-choice (MC), multi-response(MR), numerical input (NI), word input (WI) and true/false/undecidable (TFU) question types. Whilst this does not allow input of mathematical expression, it is very robust and care has been taken to avoid frustrating the student with complex input syntax. For numerical input questions, sympathetic handling of decimals is needed if not quoted to the correct accuracy, a warning and automatic correction is generally executed. For example, evaluating an integral to 2 decimal places could yield 1.20 but students should not necessarily be marked wrong if they input 1.2 or, say, 1.201742 (directly from a calculator display). Similarly, word inputs are case checked (students may have caps lock on for example).

Evidence from answer files shows (as expected) that MC questions have a far higher success rate than NI questions which are far more discerning of mastery. Depending on the purpose of the test, this may, or may not, be a good thing; starting students off with largely MC questions can and does boost their confidence enough to tackle NI presented later. By programming mal-rules (see e.g.[10]), both MC questions and responsive numerical input (RNI) questions can provide feedback tailored to many of the student’s responses. TFU questions are particularly suited to examining student’s understanding of concepts rather than manipulation skills (see Fig 2).

Fig 2 – Randomly-chosen subjects (equations) have been spliced with randomly-chosen properties (words) to create a large number of statements. To mitigate against the effects of guessing, all answers must be correct for a mark. Note: the colours and fonts are under the student's control, thus enhancing accessibility.

Obviously, these objective tests must be embedded within a module syllabus and the overall course curriculum. The downloads do not write answer files and can be used purely as a private study resource; in fact, students spend most of their time carefully studying the feedback, often after deliberately inputting a random response simply to access the feedback. Thus Mathletics should properly be seen primarily as a learning resource and less as an assessment engine. Of course, some (many/most?) students will want marks, perhaps as a hedge against anticipated poor exam performance. Short of invigilation of the test at specific times, how then can one avoid cheating by aliasing (e.g. “I’ll do your maths test if you do my engineering drawing”), using other software (e.g. symbolic engines or Excel) or working in groups (not necessarily a bad thing)? I recommend that tests should be continuously available for private study or even for marks, but that they are strictly required to pass the unseen written examination at the
end of the module (i.e. no hedge is possible). Thus a student who has cheated will not pass unless they have somehow learned the required content in another way. This is a ‘clean’ solution that costs no additional staff time, but yet gives the incentive for committed students to improve their grades via the tests (and learning a lot by doing so).

One final comment: the online questions test highly-constrained situations with definite answers (objective questions). There is therefore no need to test these skills again (certainly not extensively) in an exam, which can therefore focus more on modelling, proof and interpretation skills that really do require a human marker and match more closely with (good) course objectives. This could mean that ‘part A’ from many exams could be dropped, shortening the exam and marking process to just ‘part B’ comprising longer questions. The effect may be that the exam itself may become harder although the overall assessment (exam plus CAA tests) should be comparable.

What I want you to do
Visit: http://people.brunel.ac.uk/~mastmmg/.
Downloads/entry.htm, download the zip files and follow the set-up instructions. When you have evaluated the tests, mount them on your Windows LAN and/or point your students at this page so they can use the resources for private study. No licence or any other software is required for fully-functioning tests (except that no answer files are written).

Give me some feedback.
If you require answer files to be written, contact me for a different version of the tests. Your Question Mark licence will need to cover your institution and the anticipated student numbers.

Tell me of additional web sites that could be linked to via the ‘Related material’ button in the feedback screens.

If you want to set up your own tests, you will need a copy of Perception 3.3 or 3.4 and the database of questions and template files containing various external functions that either return mathematical results or MathML or SVG display strings. I will give you this and talk you through how to link to the template files when creating an assessment.

If you want additional question topics and/or you can contribute additional question styles, please also contact me.

References

Notes to article
1This is not to be confused with various other search engine results on Mathletics; this yields various commercial products aimed at school children, a number problem about an athletics match and a really good song/video by the Foals that “rhythmically it does enough to draw you in and sweep you along with the punk/funk, tight drums and stabbing guitars...” – see http://www.youtube.com/watch?v=WjNNWvz3JT4.
2This is about students’ negative attitudes to mathematics (I think), but, even after reading the lyrics (http://www.lyricsandsongs.com/song/883097.html), I am unable to work out what the message actually is!