Mathematical self-efficacy and student engagement in the mathematics classroom

In recent years, the teaching of mathematics at all levels has steadily moved up the UK education agenda [4] as fewer students opt to study mathematics after the age of 16 and those involved in the provision of higher education (HE) and the employment of graduates bemoan the lack of quantitative skills possessed by employees and students. This problem is particularly acute among students who need to study mathematics as a supporting subject (for example in engineering or computing) and can be quite critical in health professions where dosage calculations and measurement are critical [5].

The issues regarding the quantitative skills gap among many university entrants and the consequent requirement for some form of mathematics support has been well documented [12, 15] and it seems that potential solutions need to go beyond the simple provision of mathematics support classes as although these can address content deficiencies, they will not necessarily help with more fundamental underlying negative beliefs and anxieties that students may have towards mathematics.

In a recent article published in MSOR Connections the influence of mathematical anxiety on performance was discussed [14]. A concept that is often considered alongside mathematical anxiety is that of mathematical self-efficacy and the latter has been the subject of much research across many subject disciplines over the last 30 years or so – particularly in the USA. In this article, we introduce the notion of mathematical self-efficacy, and discuss the link between self-efficacy and student engagement. We also look at how self-efficacy and student engagement can be enhanced drawing on some results of a small study of students studying mathematics as a supporting subject, in this case as part of a computing degree at London South Bank University (LSBU).

Mathematical self-efficacy

Self-efficacy is a type of personal cognition defined as “people's judgements of their capabilities to organise and execute courses of action required to attain designated types of performance” [2]. This concept has been applied within the field of educational research to a variety of subject domains (including mathematics) and at a variety of levels [6, 7].

An individual's judgement regarding their mathematical self-efficacy is believed to be made on the basis of four main sources of evidence. The first is performance experience which the literature suggests is one of the key sources which can inform the others. Performance experience is evidence derived from successful attainment in previous mathematical assessments. It is also sometimes denoted as subject mastery. It is very likely that success in mathematical assessments for example will
strengthen self-efficacy beliefs whereas repeated failure will weaken them. The second source of evidence is “vicarious experience” and this is obtained by comparing oneself with peers, colleagues, classmates etc. If students compare themselves against their peers and make the judgement that the comparison is favourable then this is likely to strengthen self-efficacy beliefs. On the other hand, feelings of inadequacy in comparison with peers are likely to undermine self-efficacy beliefs. The third source of evidence is “verbal persuasion”. This relates to comments made by others (usually in a position of authority such as a teacher or parent) regarding ones ability to complete mathematical tasks and also feedback received on work completed. Positive comments are also likely to be self-efficacy enhancing whereas derogatory comments are not. Finally, “physiological and affective states” are the inner feelings of anxiety, worry, tension etc. that might be provoked by having to undertake mathematical tasks. Of course, these states may also have a positive orientation when students feel happy, confident and able to perform the required mathematical tasks. Lowering levels of anxiety and stress, or increasing feelings of confidence for example, would tend to enhance self-efficacy.

**Linking self-efficacy to engagement**

A student’s level of self-efficacy is a measure of their belief that they can, in a given situation, successfully complete a particular task. The literature suggests (and is supported by empirical evidence) that self-efficacy is related to actual engagement with learning and therefore to outcome measures of learning [11]. Since the assessment of learning provides evidence through which students’ self-efficacy beliefs are nurtured, we have a classic feed-back process in operation. This is described in Fig 1.

![Fig 1 – A learning loop (Amended from [8])](image)

Linnenbrink and Pintrich [8] give a description of this feedback process and divide student engagement into three distinct types, each of which is influenced by self-efficacy. The first type is “behavioural engagement” which is the observable behaviour we see as teachers in the classroom. This relates to the efforts students are putting into mathematical tasks and how students relate to each other and to the teacher in terms of their willingness to seek help, attendance at the classes etc. Academic studies have shown that self-efficacy is related to behavioural engagement in terms of the attendance, effort, and persistence shown by students [2] and their willingness to seek help [13]. High levels of self efficacy are likely to encourage perseverance in the face of difficulties and persistence in completing a task that the student cannot immediately do. Also the response of the student to an encountered difficulty is likely to be to ask for help. Low levels of self-efficacy lead to feelings of helplessness and perhaps a premature admission of defeat. Also students with low self-efficacy are less likely to ask for help as they fear that others will interpret their difficulty as just stupidity or ignorance.

The second type of engagement is “cognitive engagement” which recognises that a student appearing to work on a mathematics problem is not necessarily indicative of the student fully engaging mental faculties in trying to complete it. Linnenbrink and Pintrich [8] describe this as students being ‘minds on’ as well as ‘hands on’! It can be more difficult to determine whether a student is cognitively engaged with material as opposed to just behaviourally engaged. In this respect, the way that classroom sessions are structured and how the teacher interacts with students is important in trying to assess cognitive engagement. Strong self-efficacy beliefs will encourage cognitive engagement. If a student believes that they can complete a task then they are likely to engage with appropriate cognitive strategies in order to complete it. Students who doubt their ability to complete a task are less likely to persist in applying cognitive and metacognitive strategies and will become disengaged if success is not immediate.

The third type of engagement is “motivational engagement”. There are many aspects to motivation but here we consider just three; the personal interest that the student has in the subject, the utility that the student feels the subject brings, and finally the general importance of the subject to longer term goals or desires. Thus, for example, a student may not particularly enjoy mathematics but appreciates the usefulness of the skills being learned and that these skills will be required within their chosen profession and so is motivated to engage in learning. There are clear links between self-efficacy and motivational engagement but there exists some debate about the direction of the cause-effect relationship. In other words, do strong self-efficacy beliefs induce greater motivational engagement or is it the motivational engagement and consequent learning that generates stronger self-efficacy beliefs? In reality there is likely to be effect in both directions and this is captured in the natural feedback process described in Fig 1.

Thus there is clear evidence of linkage between self-efficacy and engagement in learning. But what of the linkage between engagement and assessment? Here again the literature has demonstrated such links [3]. Research findings have consistently shown that student engagement is linked positively to the development of critical thinking and to
assessment results and further that appropriate design of assessment tasks can be used to encourage student engagement [9]. Finally, as we have already described, assessment results are a key source of evidence of self-efficacy and this final link completes the feedback process.

An unfortunate consequence of reinforcing feedback loops is that such a loop could be either positive or negative. In the negative case, dwindling self-efficacy can lead to reductions in engagement with learning which generates poor assessment results and this further undermines self-efficacy. If we wish to ensure that the feedback loop is a positive reinforcing loop then we need to answer three questions:

1. Are we sure that our students are aware of all the sources of evidence that inform their self-efficacy beliefs?
2. How do we observe the strength of engagement with the learning process other than by using broad-brush measures such as pass rates?
3. How can we use classroom practice and curriculum design to enhance self-efficacy and student engagement so as to generate positive reinforcement?

To answer the first of these (and part of the second) a small scale study was undertaken with a group of first year computing degree students at LSBU for whom mathematics is a compulsory first year unit. To answer the remaining questions we can turn to the literature for advice.

The study

All students at LSBU who study degree courses in either computing or business information technology are required to complete a mathematics unit as part of their first year curriculum. The content of the unit includes a range of topics in discrete mathematics (e.g. graph theory, Boolean algebra, propositional and predicate logic) as well as a number of topics from continuous mathematics (e.g. number bases, algebra, equation solving). Students are required, on entry to the course, to have GCSE mathematics (at grade C or above) or an equivalent qualification but the diverse nature of the student intake means that the mathematical skills exhibited by these students at the start of year 1 can be extremely varied. Although the pass rates for the unit have been acceptable, there are students who struggle with the material and who need additional help and support. The mathematics unit runs for an entire academic year with two hours of class contact time per week. In addition, there is already a further hour of timetabled support time which is designed to allow students time to strengthen their knowledge and skills in six basic mathematical areas. Students must attend the weekly support session until such time that they are able to pass a ‘driving test’. This is a multiple choice quiz covering the six topics that is offered every two weeks but requires a high standard to pass. Once passed, students are not required to attend the support sessions further. The driving test does not contribute towards formal unit assessment.

In September 2006, a random sample of 16 students were selected from the first year cohort of 100 or so students and each was interviewed individually. Each interview lasted approximately one hour and all were conducted by the same interviewer over a period of four weeks. The interviews were recorded and later transcribed. For the purposes of this article, all students taking part are identified only by letter to maintain confidentiality.

Each interview was semi-scripted in that the interviewer followed a protocol of prescribed main questions designed, for the purposes of this article, to explore:

1. Students’ attitudes towards the study of mathematics in terms of the relevance of mathematics to their chosen computing degree and their personal feelings towards mathematics or feelings of anxiety, if any; and,
2. To find out how students evidence their feelings of confidence in undertaking mathematical tasks, and what might enhance their levels of success.

Once the interviews had been completed the audio recordings were transcribed and then coded. Analysis of the transcripts was structured around the questions asked. For each question, the transcripts were used to identify and code common themes among students and then a second round of coding allowed the linkage of actual interview data to each of these common themes. We now discuss each of the three questions raised earlier.

“There is clear evidence of linkage between self-efficacy and engagement in learning. But what of the linkage between engagement and assessment?”

Question 1: Are we sure that our students are aware of all the sources of evidence that inform their self-efficacy beliefs?

The students were asked to describe the factors that they felt contribute to their feelings of confidence or otherwise in mathematics learning on the unit. In other words, which factors influence their judgements about their capability to complete the unit. These relate to the sources of evidence for mathematical self-efficacy and in Table 1 (overleaf) the students responses have been summarised. It is important to note here that students made no distinction between their mathematics unit and other units they were studying (they were asked specifically about both). The factors listed therefore apply equally to all units under study and testify to the wide applicability of self-efficacy concepts.

The students are listed in decreasing order of performance on the unit and what seems clear is that one of the main sources of evidence is performance experience, either in the form of the actual marks obtained on assessments or in
having a sense of understanding the material presented in the classes.

Another common source of evidence mentioned in the interviews was verbal persuasion in the form of feedback from teachers and also vicarious experience represented by comments and feedback from their peer group. These sources of evidence seem evenly distributed throughout the group of students, having been mentioned by the good students as well as the less able. The data confirm that, as a group, students of all abilities are making self-efficacy judgements based on all four sources of evidence as suggested by the literature. However, some individual students (student I for example) are only citing performance experience and for students such as these it is important that they should be encouraged to use all sources of evidence.

Question 2: How do we observe the strength of engagement with the learning process?

The approach adopted to mathematics teaching on the undergraduate computing degree is that teaching is undertaken in small group sessions (no more than 25 students to a group) using a broken lecture style in which new material is presented and discussed, worked examples are demonstrated, questions are set for students to attempt with tutor support and then students reflect on their learning so that common problems with the material may be addressed. Examples are set within the context of computing where possible and this approach has been found to provide a clear, organised learning structure for students, to encourage engagement with the material and to help students see the relevance of the mathematics they are learning. The use of a broken lecture style allows more frequent opportunities for teachers to work with students (and for students to work with each other) and the teacher can observe the extent of cognitive engagement exhibited by the students. Registers are taken regularly as one measure of behavioural engagement (non-attendees are contacted by faculty support staff) and the broken lecture style again allows the teacher to monitor behavioural engagement and, if necessary, work with students who are showing poor engagement either in class or in the support sessions.

Observing motivational engagement in the classroom is rather more difficult so this aspect was particularly addressed as part of the student interviews. Table 2 shows results from the student interviews.

Of these 16 students, six did not expect to have to study mathematics specifically on their course and four admitted to being worried about the prospect of having to do mathematics (of these four one had a strong background and one an intermediate background of mathematical study).

In terms of motivational engagement all students felt positively about at least one of the three components and
11 in at least two of the three. All students thought that mathematics was a necessary skill to be learned as part of their computing degree course. As we might expect, the students who have been less successful in their mathematical education prior to coming to university are those who tend to have the least positive motivational engagement.

Question 3: How can we use classroom practice and assessment practice to enhance self-efficacy and engagement so as to generate positive reinforcement?

In a review of self-efficacy, motivation and achievement in the development of written skills, Frank Pajares notes that some researchers have suggested that teachers should pay as much attention to the development of students’ perceptions of competence as they do to the development of actual competence. As he concludes, “… there are situations in which inaccurate self-beliefs, rather than a weak knowledge base or inadequate skills, are responsible for students short-changing themselves academically.” [10] We therefore consider some suggestions for the strengthening of self-efficacy beliefs before looking at some suggestions for enhancing engagement in the classroom.

Strengthening self-efficacy beliefs

First of all it is important to consider the four sources of evidence that students use as a basis for making self-efficacy judgements. If self-efficacy judgements are to be accurate then the learning, teaching and assessment strategy adopted needs to be such that it exposes students to all four sources at regular intervals and allows them to match their expectations of learning with actual learning outcomes. This helps in the calibration of self-efficacy and can help overcome what we have found to be a common problem among weaker students of over-estimating self-efficacy. In teaching our mathematics unit we have opted for small group sizes and a broken lecture style as a way of providing varied and continuous evidence feedback to students. The provision of regular summative and formative assessment points during the unit also provides evidence of performance experience as do the regular driving tests offered to students in their support sessions.

In addition to this [8] suggest the following:

i) The feedback provided to students is very important in helping them to maintain high and accurate self-efficacy beliefs. Feedback should be specific to particular aspects of the work being assessed (rather than just general “well done” comments) and praise the student where possible without being too positive when it is not deserved;

ii) There is always a temptation to set tasks for students that are relatively easy in the belief that this will build confidence. While this is true to a certain extent, and it certainly helps in developing a number of repetitious skills, real enhancements to self-efficacy stem from the successful completion of more difficult tasks; and,

iii) Students should be made aware that mathematical competence and ability are changeable and can

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<th>Students in decreasing order of performance</th>
<th>Did you expect to have to study maths?</th>
<th>How did it make you feel?</th>
<th>Are you interested in studying maths?</th>
<th>Is studying maths useful in computing?</th>
<th>Is studying maths useful in general?</th>
<th>Level of maths on entry</th>
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Table 2 – Student responses on motivational engagement
be developed and improved through practice and experience. Thus low self-efficacy can be improved and students should not consider themselves hopeless cases whatever their previous educational experiences.

“Finding ways to reduce anxiety and enhance self-efficacy and engagement can significantly improve student performance in mathematics and, as such, can be a powerful complementary approach to mathematics support.”

Just providing students with specific and honest feedback is not, in itself, sufficient to guarantee changes to self-efficacy. Students should be encouraged to reflect more critically on their performance, not just in mathematics but across all units that they take. When assessed work is returned to students, teachers should encourage students to read the feedback carefully and try to acknowledge their weaknesses and plan to overcome them with the teacher’s help. For our mathematics unit, this would be a helpful task for the weaker students during the support sessions. The support sessions during the first semester are targeted at the six core topics mentioned earlier, but during the second semester the students should be required to set their own support agenda according to the particular weaknesses that have been identified.

**Strengthening student engagement in the learning process**

Strengthening self-efficacy beliefs will in itself help to strengthen student engagement but there are also some useful principles that can be applied within curriculum development that can help. Myers and Nulty [9] give some practical guidance by describing five principles that they suggest will enhance student engagement. These can be combined with the earlier division of engagement types as in Table 3.

The five principles described are very much oriented around the use of assessment as a driver for cognitive engagement. It is often the case that students will, at the start of a new unit, be keen to know how the unit will be assessed as this often (rightly or wrongly) provides the scaffolding around which they organise their learning. Myers and Nulty argue that the unit assessment strategy should therefore inform curriculum development and that “… careful choice of appropriate assessment strategy (not items) can insure that the students engage…” [9].

The literature also suggests that for effective cognitive engagement students should have an awareness of critical thinking skills and learning to learn strategies. Learning to learn strategies now often form a part of first year curricula due, in part, to the diverse nature of the UK university intake and increasing interest in metacognitive strategies [16].

**Conclusion**

Finding ways to reduce anxiety and enhance self-efficacy and engagement can significantly improve student performance in mathematics and, as such, can be a powerful complementary approach to mathematics support.

Our small student survey has confirmed that on our mathematics unit students are being exposed to a variety of sources of evidence for developing accurate self-efficacy beliefs but that we could do more in some areas. For example, Table 2 indicates that the majority of our students are interested in studying mathematics but from Table 1, only one student actually enjoyed studying on the unit! Clearly we need to address this aspect of our teaching on

<table>
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<th>Components of Engagement</th>
<th>Curriculum Design Principle</th>
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<tr>
<td>Behavioural Engagement</td>
<td>Principle 2: Develop a delivery style and learning environment that is both challenging for all students (including the stronger ones) and interesting to study.</td>
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</table>
| Cognitive Engagement     | Principle 3: Try to design an assessment rationale that obliges students to engage with the learning process and which requires deep rather than surface learning.  
                          | Principle 4: Try to design an assessment rationale that is realistic (i.e. contextualised to computing in our case), in which the assessment components are linked to each other and cumulative in terms of the knowledge and skills to be applied. |
| Motivational Engagement  | Principle 1: Emphasise the real world relevance of the knowledge and skills developed in the unit at the start of the unit. How does the unit relate to the study of computing and to the world of work in general?  
                          | Principle 5: Make clear connections between all elements of the unit so that students can appreciate that, as a whole, what they have learned will have practical and real-world applications. This is the embodiment of Principle 1. |

Table 3 - Student engagement principles [9]
the unit if we are to improve the physiological and affective states evidence base of mathematical self-efficacy.

We also need to do more to encourage student engagement. The interviews indicate that we seem to be relatively successful in terms of motivational engagement but classroom experience is that we are sometimes less successful in terms of behavioural and cognitive engagement. Having developed an effective delivery model for the unit, we will now be looking at how we can better use assessment methods congruent to the five principles to encourage these aspects of student engagement.

References