Introduction

In level 3 of modular courses it can sometimes be difficult to provide students with a full range of modules to meet all needs. It is common practice to offer “independent study modules” to cater for this and to allow students to pursue any particular interests they may have. These allow students to pursue their interests with only a minimal amount of staff input. (Note that the word “independent” indicates this relative lack of structured staff input – there may be two or more students working together on the same topic.)

Our experience of such modules is that they can be problematical – on occasions they can end up consuming more staff time than had been anticipated, and can result in students not achieving appropriate learning outcomes.

Undergraduate students are often neither good at organising their own learning, nor at the self-analysis which is needed for them to become effective learners. This contrasts with the approach within, say, the teacher education community, where the model of “the reflective practitioner” informs the learning process.

The project

This paper reports on a University of Brighton project, part funded by the Maths, Stats and OR Network, to improve the quality of individualised learning in the final year of the university’s mathematics degrees.

The aim of the project was to identify and compare methods by which students can more effectively learn a mathematical topic which is new to them, through the medium of an independent study module. The process was intended, in itself, to foster the acquisition of transferable learning skills.

The project had two phases, the design and validation of three specific independent study modules, and then their delivery. The first phase was achieved during the first part of 2007, so that the modules were available for level 3 mathematics students in the academic year 2007/8. Phase 2 was dependent on student demand. In the event only one student chose an independent study module. This report is therefore limited in its scope in terms of analysing the desired final output – better educated undergraduates.

The modules

Three independent study modules were written and validated. They were in topics in which expertise was available, but for which there has not been sufficient demand to make them available in the usual undergraduate programme. The topics were Number
Theory, Topology and Complex Analysis. The contents of these modules are standard, but the aims, objectives and methods of assessment were designed to be consonant with delivery through independent study. The module specifications may be seen at:

http://www.cmis.brighton.ac.uk/staff/kp4/independentstudy/modules/modules.htm

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Some relevant parts of the Number Theory specification are produced below:

From the “Aims” section:

• to develop skills in independent study and to foster a reflective and self-analytical approach to learning

From the “Learning objectives” section:

By the end of the module the students should:

• have produced a record and an analysis of their learning process

From the “Learning strategies” section:

• Fundamentals + Prime numbers – A (fairly) traditional start. Notes will be provided, together with exercises. Students will be required to complete the exercises and to extend the notes – the notes do not cover Fermat numbers and Mersenne primes.

• Congruencies – Input from staff to start with – congruencies and modular arithmetic, including exercises. Thence web research (Wikipedia et al) for Fermat’s little theorem and Wilson’s theorem

• Cryptography – Open University material, accessed from StudentCentral


From the “Assessment” section:

The Material (60%)

• Fundamentals + Prime numbers – Short notes on Fermat numbers and Mersenne primes. An electronic diary of work undertaken – what was read when, together with an electronic sample of exercises completed.

• Congruencies – A brief comparative critique of at least three websites relating to Fermat factorisation, Fermat’s little theorem and Wilson’s theorem. Which was best for what?

• Cryptography – RSA versus Mathcad – up to what point can Mathcad crack the code? – a demonstration.

• FLT – A commentary (written or recorded) on Simon Singh’s attempt to popularise the proof, comparing his work to the proof in the special case n=4.

Reflection (40%)

• Pulling it together – what was learned and how – what was best for me. This to be encompassed in an end product, possibly video based, drawing on the above.

Summary:

The module content covers central themes in elementary number theory. The method of delivery is through guided self-study. The assessment regime is designed both to ensure that the content is learned and that independent study skills are developed. This is achieved through the requirement for reflective analysis both of the material and of the learning.

The guide

A substantial guide was produced to help students adapt to the rather different demands of these modules, compared to those of most traditional mathematics modules. This was central to the project.

It drew heavily on material from the discipline of mathematical education, wherein students are routinely immersed in:

• reflection and self-analysis

• mathematical pedagogy

• being "mathematical"

• how to do mathematics

• modelling in mathematics

The guide draws on a selection of important developments in mathematical education, past and present. It is available at

http://www.cmis.brighton.ac.uk/staff/kp4/independentstudy/Appendix1.pdf

The delivery

The one student choosing an independent study module during 2007/8 subsequently applied for and was accepted, on a PGCE Mathematics course. The student was therefore easily tuned into the reflective learning approach, and clearly benefited enormously from it.

Some extracts from the report on the student’s work are included in

http://www.cmis.brighton.ac.uk/staff/kp4/independentstudy/Appendix2.pdf
The student clearly enjoyed the module. The output produced was somewhat less multimedia than we had envisaged, but was nevertheless of good quality. In particular the summary of Simon Singh’s book was an excellent 3000 word read which was captivating in itself. The conclusion reads thus:

Simon Singh does an excellent job of popularising the proof of Fermat’s last theorem, firstly it is the story of how Andrew Wiles refused to give up on a problem that had obsessed him since childhood. The fact that the main body of work was achieved in a seven year hermit like existence, and that he thought he had proved it but was found to be mistaken, only adds to the drama. Secondly the book is interesting from a historical point of view. The discoveries of the Pythagorean brotherhood are not ‘hard’ mathematically but are remarkable when one considers the time in which they came about.

The information about Fermat provides an interesting insight into 17th century France. It is remarkable when one considers that it was through luck that the bulk of the Arithmetica survived the Christian and Muslim massacres of the first century and it was by luck that Bachet translated the text important enough to translate into Latin in a time when mathematics was not encouraged. Were it not for these two events of chance Fermat would have never read the Arithmetica. It is also lucky that Fermat’s son considered his writings important enough to publish posthumously.

The information about Germain and the other female mathematicians is intriguing from a historical and social perspective and generates much reader empathy. The inclusion of other personal stories such as the suicide of Taniyama and the untimely death of Galois generate sympathy in two ways, firstly for the individual circumstance and secondly for the loss of brilliant young mathematical minds to the mathematical community.

As a result Singh’s book can be enjoyed on many levels. There is no content from Wiles’ actual proof which is fortunate, for as Singh says, only 10% of number theorists would understand it, let alone anyone else. There is a reading list which covers things such as elliptical equations, Taniyama-Shimura, Euclid’s work and other number theory topics mentioned in a more mathematical way, that would be appropriate for university level mathematics. The appendices further explain some of the content of the book and would be understood by A-level mathematics students. In the main body of the book there is very little mathematical content – all that is really required is an understanding of Pythagoras’ theorem.

There was plenty of evidence, here and elsewhere, that our student had taken on board the essential elements of reflection – and had learned quite a lot of number theory. Indeed, drawing on past experience of other students, and of our experience over two and a half years of the development of this particular student, we felt that her learning had exceeded that which we might otherwise have expected.

Conclusion

Independent study modules occupy a small, but important part of many mathematics degree programmes. This work offers anecdotal evidence that there may be scope for improving structures, and hence student experiences and outcomes.

The last words are from the student, who attended an “Assessment for Learning” conference at the university during her studies. In her report on this she said:

Discussion about independent study modules:

It was stated that they are very narrow as [they] are just about the individual and what they want to learn. I disagreed as I feel I have learnt more in this module than in any other so far this year. I also feel it is good preparation for when I will have a really heavy work load with my project, as I am now used to working on it independently for the majority of the time. It has also broadened my learning experience as I wouldn’t have even been at the conference if I hadn’t elected to do this module. It gives me a break from the usual methods of assessment ie no exam and the exercises that I hand in are based on what I have found interesting through my own reading. It has changed my approach to learning. I get a great deal of personal feedback. I was asked if I decided any of the module objectives. I said no but they were discussed with me and if there was anything I wanted to add it would have been considered.

And finally:

In terms of workload I have spent a long time on most of the topics in this module. However I spent a lot of time because I was enjoying what I was doing and I feel that I have produced work that I am really proud of. I think in this module especially you get out what you put in. If you spend an average amount of time, you will produce average work and probably find the module reasonable. I have found it really enjoyable and hope that this is reflected in my final mark.

It was!