Mathematics, statistics and operational research (MSOR) present unique difficulties in the accessibility of content for students with visual impairments which are often little understood and so can be overlooked. This paper focuses on areas where MSOR subjects differ from general accessibility considerations, drawing on a qualitative study of staff and students from four UK universities.

An outline is given of methods of accessing MSOR content covering Braille, large print, speech, graphics, software and equipment such as scientific calculators. Accessing MSOR content in these forms causes some difficulties, particularly in reading, comprehending, manipulating and writing notations and graphics using linear and tactile formats. The conversion of materials to alternative formats can be time consuming and error prone causing difficulties to students in reading around the subject or following the course at the usual rate.

Traditional teaching methods can cause problems, particularly where the student is expected to see lecture content on a screen or board. Advance materials and care over spoken mathematics can greatly assist here. Assessment can be unusually time-consuming and extensions may be necessary, although these can cause additional stresses for students trying to complete an increased assessment load in the same time period.

Students with visual impairments can study MSOR subjects successfully provided the right support is in place. What constitutes the right support depends greatly on the student's individual needs and these must be listened to.

1. Introduction

Mathematics, statistics and operational research (MSOR) underpin all of science and engineering and form an important component in many other areas. Yet mathematics and statistics present unique difficulties in the accessibility of content for students with visual impairments which are often little understood and so can be overlooked. This paper focuses on areas where MSOR subjects differ from general accessibility considerations. Good guides to general accessibility exist and are worth consulting (see [1], [2]). Students with visual impairments have issues in numerous areas of MSOR subjects but the main problems are reading, writing and manipulating mathematical notation, accessing graphics, using equipment (e.g. calculators) and mathematical software. Students and staff employ a number of different approaches, some of which are described in this paper.

The way in which a course is operated can have a dramatic effect on the experience of students with visual impairments. There is a firm belief that issues of access to MSOR courses can be overcome and students with visual impairments can successfully study, with the right support and adaptions (see, for example, [3]-[6]). There are legal (in the United Kingdom, for example, see [7]-[9]) and moral obligations to adapt courses for students with visual impairments as well as financial considerations attached to the widening participation agenda. In addition, the flexibility implied by an approach to supporting students with visual impairments has pedagogic benefits to all students.
To inform this research, data collection took the form of face to face, semi-structured interviews conducted with staff and students from four UK universities. These universities were chosen using a purposive sampling approach whereby institutions that were thought to be particularly illuminating or interesting were selected for study. Within these institutions, gatekeepers were chosen from among one of the authors existing contacts. These identified initial potential participants and further participants were discovered using snowball sampling. Interviews were audio recorded, with consent, and anonymised written transcripts were produced. In this paper, the university names are removed and participant’s names have been replaced with self-chosen pseudonyms. Staff job titles and names of departments have also been replaced with generic equivalents.

2. Methods for accessing MSOR content

**Braille:** Some Braille systems use a cell of six dots, while some use eight dots (see Fig 1). Braille is either produced on paper using a Braille embosser or displayed using movable raised metal pins on a Braille Display, which attaches to a computer in a similar way as a keyboard. Many incompatible standards exist for Mathematical Braille and participants found students may be familiar with one particular standard, which may not be the one used at that university. Fig 2 shows an example of a linear Mathematical Braille code for a simple equation. Due to the extra control characters (capitalise, raise to power, lower again) this notation is greatly expanded.

![Fig 1 – Six- (left) and eight-dot (right) Braille cells](image)

\[ H^2 = O^2 + A^2 \]

![Fig 2 – An example of a simple equation (above) represented as Braille Authority of the UK Mathematical Braille code (below)](image)

To avoid problems of expanded notation and reduced comprehension introduced by linearity, two-dimensional Braille is sometimes used. This uses Mathematical Braille for the letters and numbers but 'draws' other mathematical symbols using Braille dots, maintaining the layout of the notation. An example can be seen in Fig 3, in which operators, the square root and fraction bar are clearly recognisable. This notation is not liked by all Braille readers. David was not convinced by two-dimensional Braille. However, he felt that if he had used this system from the start it he, “probably would get used to [the differences]”. He said that a student new to Mathematical Braille should give “good consideration” to using a two-dimensional Braille system, but be aware of its drawbacks. It is difficult to manipulate and write mathematics using two-dimensional notation.

![Fig 3 – Two-dimensional Braille representation of the quadratic formula](image)

**Large print:** The term “large print” does not simply mean enlarging the print. Most users require changes to spacing, layout, font used or colours. Sometimes in enlarging, mathematical symbols are too close to distinguish. Hazel, said: “So you have a fraction times a fraction, the spacing that LaTeX puts between those two fractions is sometimes not adequate, you can’t see that there are actually two multiplied by an invisible multiplication sign” (see Fig 4). Cahill, et al [12] report on similar problems with spacing in equations and found that students experienced less confusion if a more bracketed notation was used.

![Fig 4 - An example of mathematical notation that may be too close for large print readers to distinguish, given by Hazel](image)

**Speech:** This can be provided by software synthesis or a human reader. A synthetic voice can be difficult to concentrate on (see, e.g., [6]), while human readers are more costly and less convenient as these can only be used at prearranged times. Speech can be ambiguous; Hazel said: “Lecturers realise that sometimes they say completely different things to what they are writing down so they...
might say ‘a plus b over c’ that could be ‘a plus b all over c’ or that could be ‘a plus stop b over c’.” Meeham, Hoffert and Hoffert [5] warn that “neither the teacher nor the student may be aware at the time that there are two ways to interpret the verbal description” (p. 243).

**General issues on linear formats:** It can be difficult to communicate MSOR materials in an unambiguous way in linear form. If mathematics is accessed in non-visual or linear format this will be more difficult to read and understand, manipulate and communicate the answer. Reading an equation in a linear format or listening to it is not as straightforward to understand as simply looking at an equation. An important component of understanding a mathematical expression or diagram is that an overview can be achieved prior to studying the detail. In the case of linear representations this can be impossible, and in the case of large print users with limited field of vision this overview may be extremely difficult if not impossible. Manipulation of equations in a linear format can be conceptually difficult, error prone and the student may find it difficult to translate the answer into a format which can be read by the assessor.

**Graphics:** Access to complex graphics can be difficult. Most participants used a mixture of describing graphs and providing tactile diagrams, depending on complexity, although in different circumstances depending on student preference. David had diagrams described if they were “simple enough;” while Eric produced descriptions as an alternative to tactile diagram on “only very rare occasions where the diagram would be so complex.” Jim worried that a description of a diagram would not be useful, as this is someone else’s interpretation.

For tactile graphics, these can be drawn or printed on ‘swell paper,’ and heat is applied to make the ink rise. Producing tactile or large print graphs and diagrams often involves simplifying the original graph or diagram first as only limited information can be presented at one time. Sarah found complex graphs may need to be plotted onto several tactile diagrams as features could be difficult to distinguish.

**Software:** MSOR subjects make use of some specialist software which is not always accessible. Where software was not accessible to assistive technologies, some students work with a sighted reader who will read the screen and operate the software. The output from some software can be converted into a format the student can access, for example textual data from statistical packages. Sometimes an alternative to standard software might be accessible and it is important such alternatives are permitted.

**Equipment:** It is important to remember that as well as mathematical content, practicing mathematics requires certain equipment that may not be accessible. A key example is the scientific calculator with its small buttons and display. In many cases accessible versions are available, with enlarged and high definition components or speech output.

### 3. Alternative formats

In order to obtain course materials in a student’s preferred alternative format editable text is needed. Editable versions of textbooks are commonly not available from publishers. John found some success contacting authors for the original manuscripts. Material not available in editable format may need to be scanned or typed so it is important that reading lists and lecture notes are made available sufficiently in advance. There are issues surrounding the reliable translation of materials. Mathematics notation can be very prone to slight errors corrupting the meaning of an expression in an undetectable way. John considers proofreading to be “necessary”, saying, “with mathematics, as something becomes ambiguous it can be read totally wrongly and you often miss the point of what’s being said.” David was told proofreading was not practical so he felt his Braille notes could not be completely relied upon. Often the expertise in mathematical notation and specialist software needed to produce an editable copy of a mathematical text lies in the department and not with the disability service and some participants employed postgraduate students for this purpose. The issues accessing materials in alternative formats can make it difficult for students to read around the subject and this can affect their progress.

### 4. Teaching methods

“Chalk and talk” is an important tool in the teaching of MSOR subjects to the benefit of the majority of students but can be largely inaccessible to students with visual impairments. PowerPoint also has issues, particularly in conversion of mathematical content into alternative formats. Having the lecture notes in advance can make a real difference here, since these give students something to follow during the lecture. It is important that lecturers are clear as they are speaking and particularly that they do not rely on gestures, for example pointing to the screen and saying “this”: “this equation,” “this sentence,” “you can read this for yourself.” Spindler [3] expressed frustration when his student attended lectures taught by instructors who were “less than flexible in their approach to teaching” and regards such issues as “clearly a solvable problem” (p. 124). However, there are problems with too much information being read from the board, where participants reported difficulty keeping track.

There is likely to be more to a lecture than is made available in pre-prepared notes, so it can be necessary for students to be accommodated by a support worker who will take notes and assist the student in following the lecture. These notes taken in lectures can take weeks to be provided to the student in their alternative format.

Due to difficulties and potential confusion accessing conventional teaching students may benefit from additional one to one tuition. Meeham, Hoffert and Hoffert [5] recommend additional tuition as a method which
“should reveal” whether a misunderstanding has occurred communicating spoken mathematics (p. 243).

5. Assessment

Students with visual impairments might require extra time in tests and exams and extensions to coursework. A support worker may need to be present to assist. The length of time for the student to read their alternative format or have the assessment read aloud, manipulate and communicate their workings and the interaction with the support worker are reasons why extra time may be required.

It is important to consider assessment types. Multiple-choice is a difficult format to access, since this requires a lot more reading than traditional questions, yet one participant reported multiple-choice was considered an “easy” test in which students were expected to achieve high marks. Meeham, Hoffert and Hoffert [5] suggest avoiding “matching questions” as these can be “frustrating” (p. 244).

Extensions to coursework can be helpful as there are many situations outside of the control of the student where work cannot be completed on time to a suitable degree. However, university courses are generally quite full and an extension to one piece of work will likely have a knock-on effect on the next and over the course of an academic year this can have serious consequences, sometimes meaning a student will have to work through the holidays or even retake a year of their degree to accommodate all the extra time needed. Consequently, some participants tried to avoid extensions where possible, sometimes handing in incomplete work or work they considered to be below their abilities in order to meet deadlines.

6. Conclusions

Students with visual impairments may use a number of methods to access MSOR content, and may have very specific alternative format requirements. Whatever a student’s chosen method of accessing materials, problems can arise in producing this format accurately, reading it correctly, manipulating mathematics, accessing graphics and producing written work. Students may require adaptions to teaching and assessment and need alternatives to standard equipment and software that are more accessible than those generally available to students. Despite all this, evidence suggests that students with visual impairments do generally do well in MSOR subjects if the right support is in place.

A cautionary note: Supporting students with disabilities means accommodating individual needs. Almost the worst outcome from you reading this paper is that you go away thinking that you know how to support students with visual impairments. It is quite normal to find two students with very similar impairments yet very different working practices. Most important is to be as flexible as you can and responsive to the individual needs of your students. The best way to find out what a student’s individual needs are is to ask them; they are usually the best expert on adapting the course for them.

References

8. Special Educational Needs and Disability Act 2001 (c. 10).

Notes to article

1 In Braille representations in this paper, a large dot represents a raised Braille dot while a small dot represents the absence of such a dot.