This article is about an MSOR Network-funded project with the same title. I describe the construction and initial testing of a DVD resource designed to help mathematicians learn more about their students’ reasoning at the transition-to-proof level. The DVD uses specially annotated video data, together with screens of prompts for reflection, to encourage discussion among mathematicians about students’ difficulties and successes. I describe part of the content, indicating how a short episode can raise many of the major issues that have been investigated and discussed in the mathematics education literature. I also report on design issues in the structure of the DVD and on mathematicians’ responses to an early version of the content. Finally, I describe the progress of the project and outline how it might be used in training for new lecturers and for graduate students involved in teaching.

**DVD purpose and structure**

While many mathematicians are interested in issues in teaching and learning, few have time to read education research papers. Further, as a mathematics education researcher, I am regularly told that those who attempt to do so often give up in frustration because it is difficult for a novice with limited time to get past the jargon of another academic field. However, as a teacher of undergraduate mathematics, I know how much I benefit both from my knowledge of this field and from my own experience of conducting research. In particular, through the process of conducting and analysing research interviews, I have learned enormous amounts about students’ general reasoning patterns and their particular mistakes and misconceptions in various mathematical topics. I find this process fascinating, and while conducting a particularly illuminating set of research interviews two years ago I became convinced that mathematicians would likely benefit more from access to the process of mathematics education research than from attempts to read its products.

As a result, and in response to an MSOR Network call for projects that would support those teaching proof to specialist undergraduate mathematics students, I began a project that aims to provide this access via a DVD resource in which mathematicians can watch individual students engage with proof-based mathematics problems. Video episodes have been edited so that half of the screen shows the student working and the other half shows subtitles at the bottom in blue and the student’s written work in large print in black at the top (a screen shot is provided in Fig 1). Both the subtitles and written work change in real time, enabling the viewer to follow the student’s thinking as easily as possible. The content of each episode is divided into 2-3-minute segments, after each of which the viewer sees a screen of questions designed to prompt reflection on what has been seen. In this way, mathematicians are invited to analyse this data for themselves, attempting to accurately characterise what they see and debating the learning and
teaching issues that arise. (For another project with a similar ethos, see [13]).

Fig 1 - Screen shot from the DVD.

**DVD content**

The data video used for the DVD was gathered at a large state university in the USA. The participants were all enrolled in a transition-to-proof course called “Introduction to mathematical reasoning”, in which they studied methods of proof in the context of abstract topics such as sets, functions and number theory. Each participant was interviewed individually, and during the interview was asked to attempt three tasks. Each task was given on a piece of paper; the student was told they could write on the paper and was asked to describe their thinking out loud. As interviewer, I remained silent except to remind the participant of this request and to clarify the nature of the task. I do not wish to pre-empt the experience for anyone who might use the DVD, but to give a flavour of the content, below is one of the tasks that we see two students attempt.

Let $A \subseteq \mathbb{R}$ be a nonempty set. $U$ is an upper bound for $A$ if and only if $\forall a \in A, \ a \leq u$.

**Task:** Suppose that $A, B \subseteq \mathbb{R}$ are non-empty sets, $u$ is an upper bound for $A$ and $v$ is an upper bound for $B$. Prove or disprove:

- $u + v$ is an upper bound for $A \cup B$.
- $u \cdot v$ is an upper bound for $A \cup B$.
- $u - v$ is an upper bound for $A \cap B$.

The reader will note that all three statements are false, so that in each case a “disproof” is required and hence a counter-example is needed. This makes the task different from those that most students will encounter in such courses. This was a deliberate research design choice, since I wanted to avoid questions that could be answered using a standard procedure as well as to examine how readily and how effectively students would use examples in their reasoning.

The data was originally intended for research purposes only, so the content that could be used for creating the DVD was constrained by the limited number of students who agreed to allow their interviews to be used for this purpose. Fortunately, two of these in particular (given the pseudonyms Kim and Nick) were articulate in expressing their thinking and made good progress on the interview tasks, without producing perfect solutions and without being maximally efficient. In my view, this type of data is particularly suited to this purpose, because it can provide valuable insight into what might reasonably be expected from a competent student and what we might therefore realistically hope to teach our students to do at this level.

**Issues and ideas from mathematics education**

The aim of the project is to allow viewers to examine and develop their own conceptions and theories of learning by discussing these with others, thereby creating, at least on a temporary basis, what Jaworski might call a community of inquiry [10]. It is worth noting that this opportunity is often lacking in the UK academic experience; in most universities, a single course is taught by a single lecturer, so there is little in the way of natural occasion for in-depth discussion of actual student responses to particular tasks or types of instruction. One thing this video material does is to provide common experience to a group which can be discussed there and then.

I consider this freedom to describe and analyse the video in whatever way seems appropriate to be an important characteristic of this resource, because it avoids telling anyone what to think (this is virtually impossible in a written piece, in which coherent presentation demands that the author impose their own logic and analysis) and therefore allows viewers more space to relate the material to their own individual teaching experience. Nevertheless, it seems appropriate here to describe briefly some issues from the mathematics education literature which I believe are raised by the video content.

- **Definitions:** We see much that pertains to students’ understanding of abstract mathematical concepts and the way in which these must relate to definitions. In particular, we see students making extra assumptions that do not follow logically from given definitions (e.g. implicitly assuming that an upper bound should be positive or should be in a set) and struggling to translate an informal argument into one based on definitions, even when these are given in a task (cf. [1, 6, 12, 19, 20]).

- **Examples:** We see the extent to which students are inclined to introduce examples and are able to use these effectively to assist them in their reasoning (cf. [2, 4, 7, 24]). We also see students give arguments that do not work for some specific cases (e.g. for negative upper bounds), which raises issues about students’ understanding of the range of examples to which a statement might apply (cf. [3, 21]).

- **Proof and logic:** We see students attempt to work with logical statements involving quantifiers and implications, and the consequences for both their conclusions and their ability to validate their own arguments (cf. [17, 18]). We
also gain insight into the students’ strategies for proving and how well developed these are at this stage (cf. (2, 22)).

- **Conceptions of proof:** We see material that raises the issue of the students’ understandings about what constitutes a proof at this level, and how this is influenced by their experience of lectures and homework in such a course (cf. (5, 14, 15, 16)). In particular, we see some uncertainty regarding what constitutes a counter-example and the relationship between counter-examples and proof [8]. This naturally raises issues of what mathematicians do in their own reasoning and how this corresponds to what is expected of students at this level (cf. ([9, 23]).

**Design of the DVD and early responses**

Two episodes were completed in 2006 and have now been presented at universities in the UK, Ireland and the USA, to groups of between 10 and 50 mathematicians. In these presentations, I showed video segments one at a time, allowing time for participants to discuss each one and guiding a general discussion to varying degrees. Here I offer some observations from this experience.

The first thing to note is that the divided-screen format has proved extremely successful in allowing viewers to follow the students’ reasoning. No-one in any of the presentations remarked upon it, and those who were specifically asked all said that they could follow it perfectly. A second point is that I realised, while presenting the material, that by splitting it into the obvious segments, I had unintentionally created “cliff-hangers”: each segment ends with some kind of error or ambiguity in the student’s thinking, which is then resolved to some extent as they take a different approach in the next segment. This is obviously not a necessary feature for such a presentation, but with hindsight it proved effective in two ways. First, having unresolved errors or ambiguities at breaks in the video seemed to provide the impetus for serious debates among the mathematicians about the precise nature of the errors, the possible sources of these and the way in which such errors might be resolved or avoided. Second, it gave the presentations a sense of drama – for most viewers there seemed to be strong sense of resolution and relief at various points. Given my goal of making a resource that would be engaging and enjoyable, this is obviously a plus.

Regarding the learning and teaching content of the material, I was pleased to find that the video-based presentation generated much discussion about the issues noted above. Many of the mathematicians present were keen to continue the discussions with me and with each other, and to hear more about the ways in which these issues are conceptualised within the mathematics education community. To give a flavour of the discussions and of the ideas that stayed with the mathematicians after these presentations, I give here some comments that were emailed to me in response to a request for feedback after I gave a presentation at the 3rd Irish Symposium on Undergraduate Mathematics Education. I think that the last, in particular, illustrates the way in which this material can contribute to positive discussion of issues in learning and teaching.

“The time flew by! It was interesting to see how other educators / mathematicians viewed the students. Some were appalled at the imprecision of students’ answers and some were sympathetic to the students’ good thinking patterns but poor formal communication of mathematical ideas.”

“Since critical thinking is a highly lauded objective for students taking maths courses, any experiment like yours which shows the evolution of the students’ understanding and tactics to solve a problem is very useful. It also helps us to see how crucial it is that students really appreciate the meaning of examples, or what the concepts really are.”

“(The interviewee) seemed very confident with his answers, yet as soon as he had given them asked the interviewer if they were correct. As mathematicians, we consider it important not just to be correct but being certain of that. For example, we use this skill any time we read a proof and so convince ourselves of the veracity of a theorem. We expect this skill to be developed, but rarely to try to nurture it explicitly.”

“I was surprised by how much progress they made on the problems when LEFT ALONE to get on with it. I know that if I was the interviewer, I would have stopped and corrected almost the second they made a mistake. It was instinctual and it surprised me! I didn’t know I was so bad. They most likely would have got the answer much quicker but would they have understood it? I learned that I need to give students a lot more time to work through what I feel are simple tasks.”

“Sometimes when a group of mathematicians gets together to talk about teaching it can easily spiral into a bit of a whinge-fest about all the things that students don’t do, or do wrong. But the fact that the two students were clearly trying so hard and were so very earnestly working on the problems, you just couldn’t whinge about their attempts. Instead you found yourself looking for what was good in their thinking and really rooting for them! And I think this was one lesson I learned from the presentation – even if a student gets something wrong, try to see what is good about their thinking and encourage that.”

**Progress and plans**

At present, two graduate students at the university of Essex are completing the transcription and editing of two more episodes showing Nick and Kim each working on a different problem, as well as some of their general comments about their own reasoning. Staff in the University’s Audio Visual Media Services have been extremely helpful in teaching us to use the appropriate video editing software, and will next help me to make a final version of the material in a format that can readily be watched using a standard player. In the next few months I will also record an introduction to the DVD and write accompanying material containing suggestions for its use and for follow-up reading by both individuals and groups.

I hope that this type of material will come to be considered ideal for use in the training of both graduate teaching assistants and new mathematics lecturers. In particular,
in light of comments about the limitations of Certificates in Higher Education Practice [11], it seems that there is a need for subject-specific materials to complement what can currently be offered through generic courses on teaching and learning. My colleague Matthew Inglis and I are currently looking into designing a project that will evaluate the use of the DVD for this purpose. In the longer term, I hope to be able to produce a small library of similar materials on a variety of topics, with a view to helping those who are teaching a course for the first time to begin their planning with some knowledge of the support their students are likely to need.

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References