ESUM (Engineering Students Understanding Mathematics) is a developmental research project at a UK university. The motivating aim is that engineering students should develop a more conceptual understanding of mathematics through their participation in an innovation in teaching. A small research team (the authors) has both studied and contributed to innovation which included small group activity, a variety of forms of questioning, an assessed group project and use of the GeoGebra medium for exploring functions. The main study took place in the academic year 2010-11, but development is ongoing.

Background to ESUM

A mathematics module for Materials Engineering students in a UK university has run for three years over two semesters with the same lecturers, a different one in each semester. In ESUM we focus on the first semester in which the lecturer has modified teaching each year, intending to create a more student-participative approach and encourage students to develop more conceptually-based understandings of mathematics. Modifications in previous years have had limited success [2] and the innovation in the third year was designed to be more coherent and far-reaching encompassing changes to how the module was delivered and the ways in which students interacted with the mathematics, the lecturer and each other. Innovation was undertaken by a research team of three teachers of mathematics (two with extensive experience of teaching engineering students and one, the lecturer, with extensive experience of mathematics teaching and teacher education at secondary level) who designed, conducted and reflected on teaching (the insiders), and a research officer (outsider) who collected and analysed data as agreed with the teaching colleagues.

The module was taught by one of the team (the lecturer) over 13 weeks with two lectures and one tutorial per week. The cohort of 48 students mostly had A-level mathematics with grades A to C, with just a few alternative qualifications; two students had no mathematics since GCSE. Lectures were timetabled in tiered lecture theatres. The weekly tutorial was held in a large computer laboratory with individual computer tables in squares of four, each set of tables accommodating one group of students. For the tutorials, students were grouped in threes and fours and expected to work together on set tasks and an assessed project. Tasks and project were designed for the module by the teaching team and formed a part of the innovation; both included inquiry-based questions designed to encourage exploration in mathematics using GeoGebra². In addition, inquiry based questions were used in lectures along with more

¹This report is necessarily short. See Jaworski & Matthews, in press [1], for further details.
²GeoGebra: http://www.geogebra.org/cms/
traditional questions to encourage student involvement and provide feedback on understanding. Question design drew on a range of published resources.

**Developmental Research in ESUM**

The project had four phases: a design phase (of questions and tasks) preceded teaching and continued in parallel with the teaching phase of 13 weeks; a phase of data analysis followed involving the research officer and two members of the teaching team; a fourth phase (which is still ongoing) involves dissemination of findings and their use in the (re)design of the module for the subsequent year which is now underway.

Research was designed both to promote development and to study it [3]. Promotion was achieved by feeding back to teaching as data was collected and by creating an inquiry approach to teaching. The research studied the entire process through a rigorous analysis of data collected. The lecturer acted as a practitioner-researcher, reflecting on all activity and feeding back from observations and other data to ongoing teaching design and practice. The outsider researcher observed lectures and tutorials, with audio-recordings of lectures. She designed and administered two questionnaires for student data and feedback from teaching sessions and, with another member of the team, held one-one and focus group interviews with students at the end of the teaching semester. All research instruments and activity were agreed first with the teaching team.

**Research Findings**

Findings have come from both insider and outsider analyses. Insider analysis has involved reflective consideration of day to day activity, often with feedback to ongoing practice. Outsider analysis has involved a rigorous analysis of data collected from events. We summarise here the key findings of the project.

Compared with previous cohorts we have observed a much greater engagement in lectures. Inquiry-based questions have contributed to this, and the lecturer’s deliberate use of a questioning approach has resulted in a much increased level of response from students (over previous cohorts). The lecturer in the second semester found remarkable responsiveness (over previous years), and the students’ lecturer at the start of their second year has remarked similarly. Marks in tests and examinations were at a higher level (approximately 10%) than in previous years. Of course, this might have been a specially responsive and able group of students compared to previous years. We were not able to compare intake qualifications since the data from previous years was not available.

Students held mixed views about the values of the innovation to them. Analysis of focus group data revealed that:

1. While they could see that GeoGebra helped them to consider a wider range of functions and fit functions to data (as evidenced by their project reports), the need to draw by hand in an examination required more practice of graph sketching without GeoGebra. Dynamic use of GeoGebra in lectures was seen as sometimes interrupting lecture flow and taking unnecessary time.

2. Computer-based work in tutorials led to temptations to engage with social networking sites. Students would have preferred more opportunity to practice solving test or exam style questions.

3. Group activity worked well for some groups but was problematic when some students contributed little to the group. It was extremely valuable for some of the weaker students when their peers helped them to understand mathematical ideas. Some students felt that the group project could have been more demanding.

4. Students felt that too much time at the beginning of the module was spent on functions which they ‘knew’ already from A-level work. They would have preferred to spend more time on matrices and complex numbers which were relatively new for many of them.

It seemed clear that students came to the module with both traditional views on learning mathematics (e.g., wanting clear explanations and opportunity for practice) and a strategic approach to their studies (what is needed to pass the exam). We set out to create opportunity for more conceptual understanding of mathematics. We observed greater engagement and higher summative results than with previous cohorts. However, we were not able to measure conceptual understanding directly through our approach. An instrument to achieve this is being trialled with the new cohort.

Student perspectives are being fed into the new design: for example, starting the year with a focus on matrices and including practice-based work in tutorials alongside exploration with GeoGebra. The group project is being redesigned. The overwhelmingly positive outcome from ESUM is what we have learned as teachers about what engaged students and how they experienced the innovative approaches we have used. We are becoming more knowledgeable about the balance of activity, about specific elements of innovation and about the issues in developing conceptual understanding of mathematics. These feed into the overall module design; they also inform day to day practice in interacting with students and discerning their quality of understanding.

**References**

