A principled approach to teaching mathematics in higher education

William Cox
Information Engineering
Aston University
w.cox@aston.ac.uk

Introduction

In the development of its support for new mathematics lecturers the Maths, Stats & OR Network (MSOR Network) has for some time been developing materials that provide an introduction to teaching mathematics in HE [3]. An important question that has emerged during this is the pedagogical level on which to base such materials. The Higher Education Academy (HEA), in its Professional Standards Framework, suggests that CPD should be underpinned by an understanding of how students learn, and most generic programmes for the training of lecturers will contain some study of such ideas. In Mathematics we are fortunate in that we have a long established discipline of Mathematical Education on which to draw – a great deal of work has been done at the Higher Education (HE) level. But HE mathematics teachers have little time for the study of pedagogy, and find it difficult enough just keeping up with the demands of day to day class contact. So, how can we train, educate and support lecturers for their teaching, while recognising at the same time the practical and political realities within which we all have to work?

At the one extreme we have the ‘teaching tips’ approach that trivializes and encourages a perfunctory approach to teaching. At the other we have the sophisticated detailed theoretical approach of Mathematical Education, which is all but impenetrable for most practitioners. We need to steer a middle course balancing theory and practice. To this end I here suggest eleven principles of teaching that are evidence-based and can be used as a basis for good teaching practice. The term ‘principle’ is used loosely here. While not having the rigour of, for example, Newton’s Laws (We are probably barely at the Kepler stage in the development of a ‘science’ of teaching.), they are however meant to be founded on convincing evidence which will be a composite of findings from Mathematics Education, widespread practitioner consensus and plain common sense. They are intended to be used as guidance for actual practice in the classroom, and so they are framed at this level. Some of the principles may be debatable, they may be incomplete, they may be ill-informed, they may be unfashionable. But, they are there for anyone so inclined to critique and hopefully improve. There is always the danger that such principles will appear bland and anodyne. But this is often the case with fundamental principles of any kind. Such principles only really come to life when they are put together, their consequences explored, and they are applied to particular situations.

The impetus for this article came from a recent meeting of mathematicians and mathematics educators at the Warwick University Mathematics Institute entitled “Mathematicians and Mathematics Educationalists: Can we collaborate?”. This was...
jointly organised by the Warwick Mathematical Institute and the MSOR Network. The meeting stimulated ideas and resources related to how the work of mathematical educationalists could be incorporated in practical teaching. A report on this meeting can be found at http://www.maths.warwick.ac.uk/~mond/mvme.html, along with contributions, including a fuller version of this article. Further contributions to this resource are very welcome.

Basic principles of teaching mathematics in higher education

Our problem is to find ways to encourage deep thinking about teaching within the competing priorities that lecturers have these days. Rather than overload the lecturer with copious disconnected and unsupported ‘tips’, we adopt the approach of providing a small number of basic principles that underpin everything we do in teaching mathematics. The hope is that in observing and embedding these principles in all that they do, the lecturer will have a firm foundation on which to build their day to day practice. The principles have been grouped under the practicalities of setting up the learning environment, how we think students learn and the main teachers’ tasks in helping them to do so.

Practicalities of providing the learning environment

These principles relate to the practical aspects of setting up the sort of environment that is most conducive to learning. Of course, this involves a large range of considerations from using accommodation and equipment effectively to building up beneficial rapport with the students. We have to think about resources available and how to use them, we need to be prepared for the human aspects of teaching, everyone involved needs to be clear about what we are trying to achieve, and the teaching, learning and assessment activity designed to do this.

P1. Resources: Teaching and learning is constrained by limited resources (the most important of which is time) available to you and your students.

While including the obvious hardware and software, it also includes the intellectual resources of our students – and ourselves. For example, some innovations in teaching are invariably labour intensive and require a lot of intellectual input, costs which may outweigh the benefits.

P2. Professional and human: Teaching and learning has both professional and human/pastoral dimensions and management of the curriculum, the student group and the associated interpersonal interactions requires a mix of both.

Teaching and learning involves continual human interaction with all that brings. The teacher has to have the detachment to deal professionally with the problems that will arise from this. And, as in most human situations, we may have to adapt our personality in order to do a good job. In our normal mathematics activities our personalities, human frailties, and emotions are not really that important – the problem and its solution will never go away, will never change and is not the slightest bit influenced by our personal emotions and foibles. However, in teaching our behaviour can have a lasting influence on our students and affect our ability to support them. A mistake in this respect may not only change the problem, but also destroy any chance of a solution. Conversely, if we get it right and establish good relations with our students then we can have a powerful positive effect on their learning, which can often transcend our professional and technical skills.

P3. Clarity and precision: There needs to be clarity and precision about what is expected of the students and how that will be measured.

We cannot get the best out of our students without being clear and precise about what we expect of them whether in overall course descriptions and objectives, or in day to day lectures and assessment. But while one aims to help students to learn in the most efficient way, so they can move more quickly onto new things, there is a danger in being over prescriptive, and striking the right balance is a difficult part of the job.

P4. Alignment: The teaching, learning and assessment strategies need to be aligned with what is expected of the students.

For given learning objectives, the teaching and learning strategies and activities must be designed to achieve them. Quite simply, students must be assessed on what and on how they are taught. The assessment strategy and tasks must be designed to measure achievement of those objectives. This alignment of the objectives, the teaching activities and the assessment involves classifying cognitive skills by some taxonomy, designing teaching methods appropriate to these skills and assessment methods to assess them. Most such taxonomies are too far removed from how practitioners operate, although there are user-friendly versions for the mathematician [2].

How students learn

We know a great deal about student learning, the real problem is that there is too much for the busy academic to assimilate and incorporate into their day to day teaching [4]. Here we try to encapsulate some current thinking on student learning of mathematics in just a few principles, at the risk of trivializing such work, but with the intention of provoking debate and encouraging lecturers to embed at least some of the key ideas from Mathematics Education into their everyday teaching practice.

P5. Intellectual load: The workload on students, in terms of intellectual progression, must be appropriate to the level and standards of the course, and the background of the students.
This apparently obvious idea is one that often gives new lecturers problems. To measure the appropriate load we need an accurate assessment of the students' backgrounds. Deciding on the appropriate workload for different cohorts of students, and detaching that consideration from confusions over ‘standards’ is one of the more difficult aspects of teaching.

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P6. Process to product: Mathematics is most effectively studied in the way it is done, rather than in the way it is finally presented.

It has long been known that the polished ‘Definitions, theorem, proof’ format in which some books and lecturers present mathematics is not how we actually do mathematics, in which the development is more exploratory, less systematic and less inexorable. It is also not how we as lecturers and researchers actually learn mathematics. This principle emphasizes that by presenting mathematics to students in a more ‘sensible’, properly explained way, leading up to a rigorous polished product, we both make it easier for them to learn and also induct them to the ethos of mathematical thinking as well as mathematical writing.

P7. Reconstruction of ideas: Mathematics is learned most effectively if the student reconstructs the ideas involved and blends them with their current (corrected!) understanding.

The evidence for this (in any subject) is clear [6,7]. Any learner absorbs the information they receive only as the first step in internalising it. It becomes material in an internal dialogue through which the learner reorganises the input and fits it into what they already know, modifying both to arrive at their own understanding. The teacher can help them in this by providing suitable activities, by intervening at appropriate points, by providing motivation, encouraging effective engagement, and explaining clearly. Mathematics is quite special in this respect. In mathematics it is probably the internal dialogue that is most important. Much of the work has to be done by the learner literally talking to themselves, and this self critical and rigorous dialogue is something that students need to develop to a high order, with our help.

The ‘corrected’ in parentheses is a reminder of the fact that the reconstruction may involve a correction or revision of previous ideas, which sometimes has to be addressed before progressing. For example, when we begin complex numbers, the student may well have had it drummed into them that they cannot take the square root of a negative number. So we may have to spend a little time explaining why it is that that is just what we are about to do. Similarly, if we are teaching completing the square we may need to be aware that some students don’t see \((a + b)^2\) as \(a^2 + 2ab + b^2\) but as \(a^2 + b^2\), which again must be corrected before proceeding. These examples illustrate that when we invite students to build on and delve into their previous knowledge, we have to ensure that that knowledge is itself secure before we start.

P8. Learning how to learn: The specific skills of learning mathematics may need to be explicitly taught.

By this we mean the skills that the student employs to monitor, adapt and apply their learning processes. That is we need to help them learn how to learn. For example they need to recognize when they need to increase their rigour, dig deeper into an issue, or check what they have done so far. The well trained mathematician takes such things for granted, but the novice may need proactive help to develop such skills. Not only that, but we need to help them to develop the skills of recognising when they need particular skills and how to set about acquiring them themselves.

Teachers’ tasks

This of course includes the usual things such as lecturing, tutoring and assessing. The technical details will be covered in training courses, mentoring, experience, and so on. But underpinning it all, we suggest there are three underlying principles the teacher needs to remember. We have to pay particular attention to how we explain things, how we engage the students in fruitful activities for learning mathematics, and how we generate enthusiasm and motivation in the students. These principles require little explanation as most experienced practitioners would take them for granted, but there is ample evidence in the literature to support them should it be needed. And each requires regular intervention from the teacher and partnership with the students.

P9. Explaining: Good skills in explanation are essential in supporting students in learning efficiently and effectively.

How well a teacher explains a topic can have a dramatic effect on how easily the student learns it. The art of good explanation is developed by practice, but the initial skills can be learnt. Effective explanation is as much about engagement and dialogue with the student as about informing them.

P10. Engaging: Students must be actively engaged in the process of doing mathematics in order to learn mathematics.

That mathematics is a ‘doing subject’ is obvious to anyone who studies mathematics, but consideration is not always given to how we get the students engaged, and with
what. If we engage then with routine, repetitive, one-step exercises, then that is what they will learn to cope with, no matter how long they spend at it. If we want to develop higher order skills in mathematics, solution of major problems etc., then that is what we have to get them working on, while at the same time providing them with the support to reach such levels.

**P11. Enthusing:** High levels of motivation and enthusiasm are required for effective learning of mathematics.

The learning of mathematics is particularly dependent on motivation. Devlin [5] argues that one of the major reasons that the lay-person finds mathematics so difficult is simply because they don't want to do it, but often have to. And even if students start off interested in mathematics, it is very easy to turn them off by not paying enough attention to keeping them motivated. Generating enthusiasm is therefore an important skill for the teacher to master [1].

**Conclusion**

An example of applying the above principles to feedback on student work is given in the extended version of this article on the Warwick website given above. In any case, the thoughtful teacher will have few problems in constructing their own examples. The main purpose of this article has been to propose an easily applicable underpinning foundation of pedagogical principles on which the teacher can base their classroom activity. A severe critique of such an approach, from both mathematicians and the mathematical educationalists would be welcome. Is such an approach useful in practice? Are the principles necessary and sufficient, is there something missing? Are they all acceptable, what is the evidence to support them?

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**References**